Cyclic modulation spectrum - an online algorithm*

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Abstract— **Many profitable methods of damage detection in rotating machines involve cyclostationarity. This approach exploits the fact that vibrations of a damaged machinery possess a periodic envelope. The frequency related to this periodic behavior is strictly related to one of the characteristic frequencies associated with design and specific operation of the considered machine. Nowadays, such methods are rarely implemented in industrial condition monitoring systems for several reasons. One of the main reasons is related to computational complexity. Algorithms that calculate cyclostationary features are said to be computationally expensive, thus their involvement in monitoring systems is limited. This drawback becomes critical in online systems in which the monitored features are updated just after arrival of the next measurement. Because of complexity of cyclostationary approach other tools are preferred instead, especially those which do not require intensive computations. In this paper we discuss algorithms for calculation of a basic cyclostationary tool, namely cyclic modulation spectrum (CMS). In this paper we provide several novel algorithms that update CMS when new measurements arrive.**

I. INTRODUCTION

Rolling elements bearing damage detection is widely studied topic and many different methods have already been introduced. The local damage of the bearing entails impulses, which can be detected in the vibration signal measured on the machine [1]. In order to indicate presence of these impulses, the vibration signal can be analyzed in several domains. A universal tool used for detection of impulses is kurtosis, which can be applied in e.g. time, frequency and time-frequency domains [2]–[5]. On the other hand, the theory of cyclostationarity can be applied to address cyclic character of this impulsive signal. Vibration responses acquired on rotating machinery (e.g. bearings and gearboxes) are considered as cyclostationary signals. The theoretical background and application methods are already well explained in the literature [6]–[10]. In [11] the Authors presented indicators which can quantify cyclostationarity of the considered signal. Moreover in [12] there are described statistical tests for testing second order cyclostationarity with non-white noise. Such tests could be applied to signals from rotating machinery in order to examine whether the machine is faulty or not. In [13], [14] the Authors propose a method for extracting second-order cyclostationary components from a vibration signal.

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This method called Averaged Instantaneous Power Spectrum (AIPS) is a time-frequency representation and its benefits are illustrated using a simulated signal, test-rig experiment and real vibration signal from a wind turbine. In [15] a blind separation problem is investigated. The Authors present how to separate two uncorrelated cyclostationary sources with the same cyclic frequency. Such problem might arise during diagnostics of closely located bearings with the same characteristic frequencies. Cyclostationarity gives also tools to model a signal from rotating machinery. For instance, periodic autoregressive time series can model a vibration signal from a planetary gearbox operating in a bucket wheel excavator [16]. Cyclostationarity of vibration signals from the bucket wheel excavator has been also investigated in [17] where a Spectral Coherence Map is proposed as a tool for fault detection. A comprehensive discussion on modeling of vibration signal from gears and bearings can be found in [18]. One of the most popular, simple and powerful cyclostationary tools is cyclic modulation spectrum (CMS). It was introduced in 2009 by Antoni [6] and described thoroughly in [19]. CMS can be considered as double DFT applied to the signal. It was developed in order to overcome the high computational cost of cyclostationary tools. Its computational cost as well as main advantages and drawbacks are discussed in [20].

However cyclostationarity based analysis is powerful, it claimed to be computational inefficient, in general. It is a crucial drawback, which impede the commercial application. There are many articles analyzing the complexity of this approach [21], [22]. In this paper we would like to propose a novel, computationally efficient online algorithm for CMS. The idea is based on the sliding discrete Fourier transform (SDFT), which was proposed in 2003 [23].

The paper is structured as follows. In Section II the introduction to CMS is presented. Then the SDFT method is described together with its computational complexity. Section IV is focused on proposing a new algorithm form CMS calculation and comparison with the classical one. Finally the conclusions are formed.

II. CYCLIC MODULATION SPECTRUM

The Cyclic modulation spectrum (CMS) was introduced in 2009 by Antoni [6]. The idea is based on double usage of the DFT to the signal. The procedure consists of two stages. Firstly, the spectrogram with N-sample long DFT from the given signal $\mathbf{X}_{0,L} = (x_0, \ldots, x_{L-1})$ with M-sample long window and S-sample step between the adjoining windows

has to be computed:

$$\mathbf{Spec}(n,f) = |STFT(n,f)|^2 =$$

$$= \left| \sum_{m=0}^{N-1} w(m)x(n+m)e^{-2i\pi fm/N} \right|^2, \quad (1)$$

where $w(\cdot)$ is a window function, $n = 1, \ldots, \lfloor (L - M + S)/S \rfloor$ is the time index, f is a frequency bin and $N \geq M$. Then let $K = \lfloor (L-M+S)/S \rfloor$ be the number of time points in which spectrogram is calculated (the number of windows). In order to compute CMS, the DFT of the spectrogram slices for each f separately has to be calculated

$$CMS(\alpha, f, \mathbf{X}_{0,L}) = DFT(\mathbf{Spec}(n,f)) =$$

$$= \sum_{n=0}^{K-1} \mathbf{Spec}(n,f)e^{-2i\pi n\alpha/K}. \quad (2)$$

In the above formula there are two different frequencies - α is called cyclic and f - carrier frequency. This procedure is one of the most useful tools in cyclostationary signal analysis. For stationary data all energy would be distributed near $\alpha = 0$ [6]. On the other hand a signal which consists of an amplitude modulated components would have cyclic modulation for $\alpha \neq 0$. The CMS is simple and provides important information about cyclostationary properties of the signal. One of the main drawbacks of this method is the upper bound for the cyclic frequency α. Indeed, there is a limitation which links carrier and modulation frequencies:

$$\alpha < f. \quad (3)$$

Moreover maximum α is equal to reciprocal of the time resolution in the spectrogram [6].

The naive algorithm for calculating CMS is via double FFT procedure. In such case the computational complexity is equal to $O(KN \log_2(KN))$. CMS for a single signal does not require time consuming computations, although naive algorithm for online CMS calculation requires one N-sample long FFT for spectrogram update and N FFT's for update of spectra corresponding to frequency bins. Thus, the time for CMS update could be longer than the sampling interval, even if CMS is being updated at every S samples arrival, for $S > 1$. Thus, it is worth to investigate an alternative algorithm which decreases the computational complexity of CMS update and makes it possible to incorporate CMS in embedded industrial condition monitoring systems.

In Fig. 1 there are presented spectrogram and CMS for a signal recorded on the faulty pulley bearing operating in a belt conveyor driving station. This signal has been already analyzed in recent papers, where several methods for local damage diagnostics have been proposed [24], [25] The sampling frequency is equal to 19200 Hz and the measurement time is 2.5 seconds. The spectrogram is obtained for Kaiser 250-sample length window with 240 overlapping samples and 1024 FFT points. One can notice several wide-band excitations on the spectrogram at carrier frequency band 1-5 kHz. These excitations are cyclic, thus wide spectral lines can be also noticed on the CMS at carrier frequencies

1-5 kHz and modulation frequency of 12.7 Hz (and its multiples). This stands for outer race local damage.
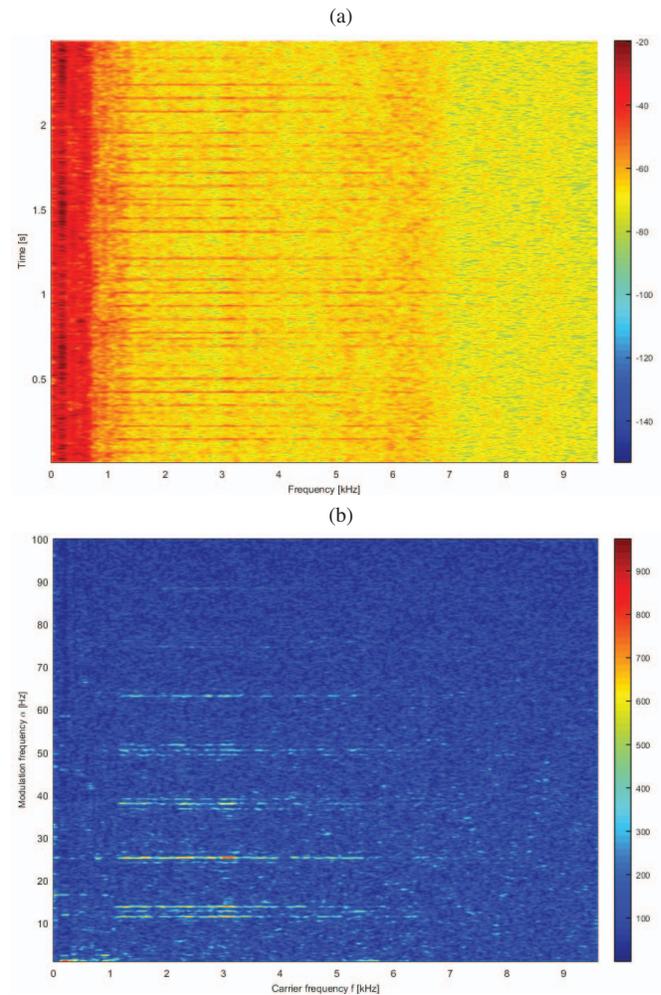

Fig. 1. The spectrogram (a) and cyclic modulation spectrum (b) generated for the signal recorded on the pulley bearing operating in a belt conveyor driving station

III. SLIDING DFT

The most popular algorithms for generating discrete Fourier Transform are undoubtedly fast Fourier Transforms (e.g. radix-2). In this section we would like to recall the Sliding DFT (SDFT) which is an online algorithm that calculates DFT of a sliding window.

Let us consider a signal $\mathbf{X}_{0,L} = (x_0, \ldots, x_{L-1})$ of length L. The classical DFT of N-sample long signal segment $\mathbf{X}_{k,N} = (x_k, \ldots, x_{k+N-1})$ for f-th harmonic bin is given by the formula:

$$\mathcal{F}(\mathbf{X}_{k,N})(f) = \sum_{n=0}^{N-1} x_{n+k}e^{-2\pi inf/N}, \quad (4)$$

where $k \geq 0$ is a window index and n is a time index. In order to calculate the DFT of the one-sample shifted window

$\mathbf{X}_{k+1,N}$ we use equation [23]:

$$\mathcal{F}(\mathbf{X}_{k+1,N})(f) = \sum_{n=0}^{N-1} x_{n+k+1} e^{-2\pi i n f/N}$$

$$\overset{[l=n+1]}{=} \sum_{l=1}^{N} x_{l+k} e^{-2\pi i (l-1) f/N}, \tag{5}$$

with substitution $l = n + 1$. Now one can add the last term separately, add the 0-th term and subtract it outside the sum operator:

$$\mathcal{F}(\mathbf{X}_{k+1,N})(f) = x_{N+k} e^{-2\pi i f(N-1)/N} - x_k e^{2\pi i f/N} +$$

$$+ \sum_{l=0}^{N-1} x_{l+k} e^{-2\pi i (l-1) f/N}. \tag{6}$$

Finally one can factor out the common term $e^{2\pi i f/N}$ and obtain formula for SDFT for f-th harmonic bin:

$$\mathcal{F}(\mathbf{X}_{k+1,N}, f) = e^{2\pi i f/N} \left(\mathcal{F}(\mathbf{X}_{k,N}, f) + x_{N+k} - x_k \right), \tag{7}$$

since f is integer and thus, $e^{-2\pi f} = 1$.

SDFT algorithm is especially beneficial in case of DFT calculation from a one-sample sliding window. The algorithm is really simple and its computational complexity is $O(N)$ for a single update, which is much lower than $O(N \log_2(N))$ for FFT. Moreover, SDFT outperforms FFT even when the spectrum is being updated at every S samples, for certain S, since SDFT for N such updates takes $O(N^2)$ and FFT - $O(N^2 \log_2(N)/S)$.

IV. ONLINE ALGORITHM FOR CMS CALCULATION

As we presented in Section III SDFT can impressively speed up the online DFT computation in specific cases. The idea of fast CMS online update is based on incorporation of SDFT algorithm. In this section we present two novel approaches to update the CMS. Both of them can give a possibility for online calculation of CMS.

Let us recall that SDFT computes DFT at a fixed number of points, i.e. size of the basis signal cannot increase. This is a crucial feature, which point out the need of analysis two separate cases. Let us consider signal $\mathbf{X}_{0,L}$ and corresponding spectrogram $\mathbf{Spec}(n, f)$ for a given spectral frequency f and time indexes $n = 0, \dots, K-1$. Then a new measurement x_L is recorded and spectrogram $\mathbf{Spec}(n, f)$ can be updated. There are two possible ways to update the CMS. The first one is to calculate $(K+1)$-sample long DFT for each frequency bin at every new spectrum arrival. Therefore the range of the signal for which CMS is calculated increases with respect to time. Namely, for a given spectral and cyclic frequencies increasing-size CMS_{IS} is given by formula:

$$CMS_{IS}(\alpha, f, \mathbf{X}_{0,L+1}) = \sum_{n=0}^{K} \mathbf{Spec}(n, f) e^{-2\pi i \alpha n/(K+1)}, \tag{8}$$

since the time axis of the updated spectrogram consist of $K + 1$ points. In the second case K-sample long DFT is

calculated for each frequency bin. Thus, the oldest spectrum of spectrogram is left out in CMS calculation when the new spectrum arrives. Indeed, for a given spectral and cyclic frequencies moving-window CMS_{MW} is defined as:

$$CMS_{MW}(\alpha, f, \mathbf{X}_{0,L+1}) = \sum_{n=1}^{K} \mathbf{Spec}(n, f) e^{-2\pi i \alpha n/K}, \tag{9}$$

since the time axis of the updated spectrogram consists of K points. Each of the case will be analyzed separately and slightly different algorithms are going to be presented for each approach.

A. Increasing-size CMS algorithm

Recall that CMS procedure consists of two steps. The first one is simply computation of the spectrogram. Let us consider STFT with rectangular window. Let us denote $\mathbf{Spec}_{K-1}$ as the spectrogram with $K-1$ time points for each of N frequency bins. Due to (1), the updated spectrogram $\mathbf{Spec}_K$ is given by the formula:

$$\mathbf{Spec}(K, f) = \left| e^{2i\pi f/N} \left[STFT(K-1, f) + \right. \right.$$

$$\left. \left. + x_L - x_{L-N-1} \right] \right|^2. \tag{10}$$

The second step of the CMS_{IS} algorithm is calculation of the DFT for each of N frequency bins. In this approach SDFT cannot be used in this step, since length of each time series corresponding to a frequency bin f is increasing in time. Thus, FFT might be used instead.

TABLE I
COMPUTATIONAL COST FOR CMS_{IS}

novel algorithm	
step	**computational cost**
STFT update	$O(N)$
K-long FFT for each f	$O(NK \log_2 K)$
final complexity	$O(N \log_2(2K^K))$
standard algorithm	
step	**computational cost**
STFT update	$O(N \log_2 N)$
K-long FFT for each f	$O(NK \log_2 K)$
final complexity	$O(N \log_2(NK^K))$

The computational complexity of each step of CMS_{IS} update is given in Table I. Moreover the novel algorithm is compared with the standard one based on double FFT usage. Clearly, the final computational complexity is lower for the CMS_{IS} with SDFT comparing to CMS_{IS} based on double FFT. The computational complexity for novel algorithm is $O(N \log_2(2K^K))$ and $O(N \log_2(NK^K))$ for the standard one. This proves the benefits of the proposed method.

B. Moving-window CMS algorithm

The proposed algorithm for CMS_{IS} is efficient, although one can look for even quicker method. The main drawback is obviously the second step, where the FFT has to be applied. In order to overcome this issue, the special case of CMS is analyzed, where the SDFT can be used for both stages.

Clearly, the first phase in CMS_MW procedure is exactly the same like in CMS_{IS}, i.e. $\mathbf{Spec}_{K-1}$ is updated to $\mathbf{Spec}_K$ using SDFT. Once the $\mathbf{Spec}_K$ is computed, the K-sample long DFT of the spectrogram (10) is calculated for each of N frequency bins. Obviously, in this approach the length of time points do not increase with respect to time. Thus, there is a possibility to apply a SDFT algorithm and CMS_{MW} is expressed by formula:

$$CMS_{MW}(\alpha, f, \mathbf{X}_{0,L+1}) = e^{2\pi i \alpha/N} \big[CMS(\alpha, \mathbf{X}_{0,L}, f) \\ -\mathbf{Spec}(0, f) + \mathbf{Spec}(K-1, f) \big] \tag{11}$$

In Table II computational cost for CMS_{MW} is presented. The novel and standard algorithms are compared. In

TABLE II
COMPUTATIONAL COST FOR CMS_{MW}

novel algorithm	
step	**computational cost**
STFT update	$O(N)$
single f CMS updatevia SDFT	$O(K)$
final complexity	$\mathbf{O(N(K+1))}$
standard algorithm	
step	**computational cost**
STFT update	$O(N \log_2 N)$
single f CMS update via FFT	$O(K \log_2 K)$
final complexity	$\mathbf{O(N(\log_2 N + K \log_2 K))}$

this approach the usage of SDFT is even more beneficial. The computation complexity for proposed method is just $O(N(K+1))$, since STFT update takes $O(N)$ and N updates of K-sample long spectra via SDFT takes $O(NK)$. Naive algorithm which involves double FFT takes finally $O(N(\log_2 N + K \log_2 K))$, i.e. $O(N \log_2 N)$ for STFT update and N K-sample long FFT's ($O(K \log_2 K)$ each). The assumption for fixed length of DFT in calculation of CMS gave a possibility for double usage of SDFT and speed up the algorithm significantly. Furthermore, it is more reasonable to assume fixed number of time points, otherwise the sample size will increase rapidly and it would be impossible to storage such amount of data. Moreover, it also affects the computational time.

V. CONCLUSIONS

In this paper the SDFT algorithm and CMS procedure were presented. Moreover, the computational efficiency of CMS was analyzed. However, the method is effective, the well known algorithms are not enough efficient for online usage. In order to overcome this issue two novel algorithms that involve SDFT for CMS update calculation have been proposed, which is the main contribution of the paper. Two different cases has been diagnosed and described. They differ in the way of CMS updating. In one case, length of DFT calculated in second step of CMS is fixed (CMS_{MW}), while in the second one it can increase in time (CMS_{IS}). For each case the novel algorithm was introduced and compared with the standard one based of double FFT. It was presented that proposed methods can significantly speed up the CMS

calculation. The smallest computational complexity was obtained for the CMS_{MW}. It can reduce complexity of update from $O(N(\log_2 N + K \log_2 K))$ in standard method to just $O(N(K+1))$. The novel update algorithm is more efficient and give a possibility for online calculation for each new recorded measurement.

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