Ref. Ares(2019)6841218 - 05/11/2019

COORDINATED PRODUCTION FOR BETTER RESOURCE EFFICIENCY

D4.2 Final report on techniques for plant-wide reactive scheduling

Lead Author: CERTH
Involved Partners: FRINSA, ASM

October 2019
www.spire2030.eu/copro

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 723575

Project Details

PROJECT TITLE	Improved energy and resource efficiency by better coordination of production in the process industries
PROJECT ACRONYM	**COPRO**
GRANT AGREEMENT NO	**723575**
INSTRUMENT	**RESEARCH AND INNOVATION ACTION**
CALL	**H2020-SPIRE-02-2016**
STARTING DATE OF PROJECT	**NOVEMBER, 1ST 2016**
PROJECT DURATION	**42 MONTHS**
PROJECT COORDINATOR (ORGANIZATION)	**PROF. SEBASTIAN ENGELL (TUDO)**

THE COPRO PROJECT

The goal of CoPro is to develop and to demonstrate methods and tools for process monitoring and optimal dynamic planning, scheduling and control of plants, industrial sites and clusters under dynamic market conditions. CoPro pays special attention to the role of operators and managers in plant-wide control solutions and to the deployment of advanced solutions in industrial sites with a heterogeneous IT environment. As the effort required for the development and maintenance of accurate plant models is the bottleneck for the development and long-term operation of advanced control and scheduling solutions, CoPro will develop methods for efficient modelling and for model quality monitoring and model adaption.

The CoPro Consortium

Participant No	Participant organisation name	Country	Organisation
1 (Coordinator)	Technische Universität Dortmund (TUDO)	DE	HES
2	INEOS Manufacturing Deutschland GmbH (INEOS)	DE	IND
3	Covestro Deutschland AG (COV)	DE	IND
4	Procter & Gamble Services Company NV (P&G)	BE	IND
5	Lenzing Aktiengesellschaft (LENZING)	AU	IND
6	Frinsa del Noroeste S.A. (Frinsa)	ES	IND
7	Universidad de Valladolid (UVA)	ES	HES
8	École Polytechnique Féderale de Lausanne (EPFL)	CH	HES
9	Ethniko Kentro Erevnas Kai Technologikis Anaptyxis (CERTH)	GR	RES
10	IIM-CSIC (CSIC)	ES	RES
11	LeiKon GmbH (LEIKON)	DE	SME
12	Process Systems Enterprise LTD (PSE)	UK	SME
13	Divis Intelligent Solutions GmbH (divis)	DE	SME
14	Argent & Waugh Ltd. (Sabisu)	UK	SME
15	ASM Soft S.L (ASM)	ES	SME
16	ORSOFT GmbH (ORS)	DE	SME
17	Inno TSD (inno)	FR	SME

Document details

Deliverable Type	**Report**
Deliverable No	**4.2**
Deliverable Title	**Final report on techniques for plant-wide reactive scheduling**
Name of Lead Partner for this Deliverable	**CERTH**
List of Authors - name(s) and organisation(s)	**Georgios Georgiadis (CERTH)** **Chrysovalantou Ziogou (CERTH)** **Michail Georgiadis (CERTH)** **Borja Mariño Pampín (FRINSA)** **Daniel Adrian Cabo (ASM)**
Version	**1.0**
Contractual Delivery Date	**31 October 2019**
Actual Delivery Date	**05 November 2019**

Dissemination level		
PU	Public	**X**
CO	Confidential, only for members of the consortium (including the Commission)	

Abstract

This deliverable presents final developments related to the activities of Task 4.1. "Plant-wide rolling horizon reactive scheduling". A novel solution strategy has been investigated for the scheduling of multi-product, multi-purpose and multi-stage production facilities as typically met in real industrial plants, similar to the use case of FRINSA. The multi-stage, multi-product facility under study consists of both continuous and batch operations resulting to an extremely complex scheduling problem. In order to reduce its computational complexity, an aggregated approach is cleverly proposed, in which the continuous processes are explicitly modelled, while valid feasibility constraints are introduced for the batch stage. Based on this approach, two MILP models are developed, using a mixed discrete-continuous time representation. All technical, operating and design constraints of the facility are considered, while salient characteristics of the canned-food industry, such as assurance of the end products' microbiological integrity, are aptly modelled. Both the minimization of makespan and changeovers is studied. In order to meet the computational limits imposed by the industry, an order-based decomposition algorithm is further investigated. Moreover, a rolling horizon algorithm is proposed for the investigation of deviations in the incoming product orders, such as, cancellation of orders, changes in order sizes or arrival of new orders. The applicability and efficiency of this solution strategy is illustrated in a case study offered by Frinsa del Noroeste. The results of this deliverable indicate clearly that the proposed modelling frameworks can: (i) be used for large-scale mixed batch and continuous scheduling problems of arbitrary complexity and (ii) provide the basis for considering uncertainty issues (e.g. in product demand) in complex large-scale problems of industrial interest, towards the development of reactive scheduling approaches.

Revision History

The following table describes the main changes done in the document since it was created.

Revision	Date	Description	Author (Organisation)
V0.1	25 September	Document created	Georgios Georgiadis (CERTH)
V0.2	25 September	Reviewed	Daniel Adrián Cabo (ASM)
V1.0	13 October	Approval by coordinator	Sebastian Engell (TUDO)

Disclaimer

Table of contents

1 Executive summary

Process industries operate in an environment characterized by increasing competitiveness and miniscule profit margins. Therefore, they must strive more than ever for efficiency and increased productivity. The decision-making process that allows for the efficient management of production and thus can directly affect the productivity of any facility is production scheduling. This process refers to the efficient allocation of resources, such as equipment, utilities and manpower, over a given time horizon of interest, e.g. daily, weekly etc., so that all required tasks are executed and incoming orders are satisfied [1]. In most industrial facilities schedules are manually generated by production engineers or operators, based on rules and heuristics, that arise from their multiyear experience and understanding of the production process. Due to the complex nature of real scheduling problems, that involve a large number of items, like tasks, intermediate and final products, multiple parallel machines, many processing stages and production routes, its computational solution becomes an extremely difficult and tedious task. Hence, numerous iterations and a significant number of working hours are required in a daily basis, which generally lead to sub-optimal results. Moreover, the generated and later executed schedules are not evaluated in terms of their efficiency, so the managers cannot assess potential real benefits realized on the plant. In this regard, computer-aided tools, that properly consider all involved parameters and constraints, e.g. operational, design, logistical etc., can assist production engineers and systematically improve their decisions [2]. Consequently, productivity can be significantly enhanced, while ensuring customers satisfaction and increase of the total profit.

Acknowledging the importance of efficient production scheduling, a number of key research contributions have been proposed to deal with the production scheduling problem over the last decades [3]. Most of these approaches consider the scheduling problem as a mixed-integer linear programming (MILP) problem. This approach has been widely preferred, due to its flexibility, rigorousness and ability to provide optimal solutions. In the early 1990s attempts have been concentrated on general representations and algorithms that cover a wide range of production scheduling problems. These efforts resulted in two general representations, the state-task-network (STN) [4] and the resource-task-network (RTN) [5]. However, the pursuit for a mathematical framework that would constitute a panacea to all scheduling problems has been quickly abandoned for representations that exploit problem-specific characteristics, due to the extremely diverse features found in the process industries (e.g. production and market environment, processing characteristics etc.). The MILP models that emerged can be classified based on the time representation, into discrete [6], [7] and continuous, which are further categorized into time-grid-based [8], [9] and precedence-based [10]. Few studies proposed mixed discrete-continuous that exploit the strengths of both representations [11], [12] .A thorough review that covers the developed MILP formulations for process scheduling can be found in [13].

Production facilities comprising of multiple batch and continuous operations, thus working in semi-continuous production mode, are abundant in the process industries. This mode is commonly used because production becomes more flexible and equipment can be more efficiently utilized, allowing the production of high-value, low-volume products. However, the scheduling problem of semi-continuous plants has received considerably lower attention compared to batch or continuous plants. Papageorgiou and Pantelides extended the STN framework and developed a mathematical formulation for the optimal campaign planning and cyclic scheduling of multipurpose semi-continuous plants [14]. In [15] the authors addressed the production scheduling problem of multiproduct multistage semi-continuous processes. Using the proposed MILP framework, they were able to solve real-life industrial study cases of an ice-cream production facility. Susarla et. al, examined the scheduling problem of semi-continuous utilizing a multi-grid approach [16]. The short-term scheduling problem of a make-and-pack production process has been addressed by baumann and Trautmann [17]. The authors developed an efficient continuous MILP model, while introducing novel symmetry-breaking constraints and pre-processing procedures that significantly improved the model performance.

The importance of applying optimization-based scheduling solutions on real-life industrial cases is widely recognized. However, only few successful industrial applications are reported, e.g. in the Dow Chemical Company [18], despite key research developments in the field of production scheduling [19]. Main reason for this disconnection between academia and industry is the fact that most contributions address small- or at best medium-sized problem instances, that do not represent to the size and complexity of real-life industrial facilities. Hence, there is a continuously growing interest in solving large scheduling problems. It must be however emphasized, that the successful use of computer-aided scheduling tools by the industrial operators and managers, is not solely dependent on the efficiency of the proposed solution strategies. Numerous practical issues need to be resolved prior to the on-site application, like ease of use, development and maintenance of the application, stable system integration, and ability to dynamically make minor adjustments and adapt to new information. In order to overcome those issues, the close collaboration of researches and industrial engineers is mandatory.

Food industrial facilities display characteristics like intermediate due dates, multiple mixed batch and continuous production stages and product quality/safety-related considerations, that substantially complicate the optimization of the scheduling decisions. The above considerations combined with market trends that impose the gradual expansion of the product portfolio, product demand profiles which are characterized by high variability and low volumes and a large number of identical production units and shared resources, make the application of optimization-based scheduling solutions in real-life industrial problems extremely challenging. Researchers have considered a plethora of industrial case studies from various subsectors of the food industry in the last decades. A real-life study case of an edible-oil deodorized industry is examined by Liu et al. [20]. In order to reduce the problem's computational complexity, the authors described the plant as a single-stage batch process, while they categorized the final products into families. Motivated by the scheduling problem of a real brewery, Kopanos et al., proposed an immediate precedence-based MILP formulation, which utilized a mixed discrete-continuous time representation [21]. The solution method explicitly models the packing stage, which constitutes the production bottleneck, while constraints are imposed for the preceding stages, to ensure the feasibility of the extracted schedules. Baldo et al. [22], considered the integrated fermentation and packing problem of a Portuguese brewery, by combining an MILP model and a relax-and-fix heuristic. In the proposed method the time horizon is split into two subperiods. In the first one all scheduling decisions are extracted, while in the second only general planning decisions are optimized. Study cases on dairy industries have motivated a number of important contributions. Kopanos et al. [23], developed an MILP model to address the scheduling problem of a real yoghurt industry. Georgiadis et al. [24], applied that model to solve the scheduling problem of a large-scale yoghurt production facility. The authors also introduced a rolling horizon algorithm to deal with uncertainty regarding new information of product orders, such as, new orders, cancellation of orders and order size modifications. Sel et al. [25], addressed the integrated planning and scheduling problem of dairy production plants, while accounting for lifetime uncertainty.

Within the scope of this work, an efficient solution strategy is proposed to address the scheduling problem of real-life multiproduct multistage process industries. A large-scale complex canned fish industry, offered by our FRINSA partner is considered as the basis for the developed modelling frameworks. The facility under study is characterized as multiproduct, multistage and consists of both batch and continuous processes. Adding to that, the large number of available units and products to be scheduled, lead to an extremely complex scheduling problem. Therefore, a novel aggregated approach is proposed to efficiently reduce the size of the problem. Based on that approach, two precedence-based models are developed, a general precedence and a unit-specific general precedence one. Both consider all industry-related constraints, thus providing feasible solutions. To satisfy the computational limitations imposed by the industry an order-based decomposition algorithm is proposed. As a result, feasible and near optimal schedules are generated for this large-scale scheduling problem. This work constitutes one of the first systematic attempts to solve the scheduling problem of a food industry of this size.

2 Description of Frinsa plant

A real-world food process industry is considered in this project. More specifically, the scheduling problem of the Spanish industry Frinsa del Noroeste S.A., one of the largest canned fish producers in Europe, is addressed. The facility under study can produce more than 400 product codes, a number that is constantly increasing, to cover market needs and fulfills more than 100 orders on a weekly basis. The production process is extremely complicated, comprising of several, batch and continuous processes. In order to simplify the description of the production process, we identify four major processing stages, in particular, thawing, filling and sealing, sterilizing and packaging, each consisting of multiple parallel units (Figure 1). Initially, the fish arrives in tracks in the form of frozen blocks, which are defrosted in the thawing stage. Then, the blocks are cut in the proper size and filled in cans along with other ingredients (e.g. tomato-sauce, oil, brine etc.) according to the recipe. In the same processing stage, the cans are sealed and transferred into carts. Afterwards, the carts are manually inserted in the sterilization retorts. To avoid the growth of bacteria, the transfer between the filling and sealing lines and the sterilization retorts must guarantee a near zero-wait policy. Therefore, no more than two hours must elapse between the completion of the filling and sealing process and the initiation of the sterilization process. The sterilization process is critical for food safety and final product quality. The cans are heated in a temperature of around 120°C, which is maintained for a specific time, that ensures the targeted bacteria lethality, and finally they are cooled down to room temperature. Depending mainly on the size and shape of the cans, but also on the type of fish and the rest of the ingredients, the duration of the sterilization process varies from 82 to 180 minutes. Horizontal retorts are used, while the temperature is managed through a water spraying system. After the completion of the sterilization process, the carts are manually extracted from the retorts and are transferred to the packaging stage, where the cans are packaged in the final product form (single, 6-pack, boxes etc.). An important operation of this stage is labeling. However, not all packaging lines have an individual labeler. In particular, lines 1-2 and 4-5-6, share the same labeling machine, therefore they cannot operate simultaneously. Finally, after the completion of the packaging stage, the end products are stored in the warehouse, to be distributed in the market.

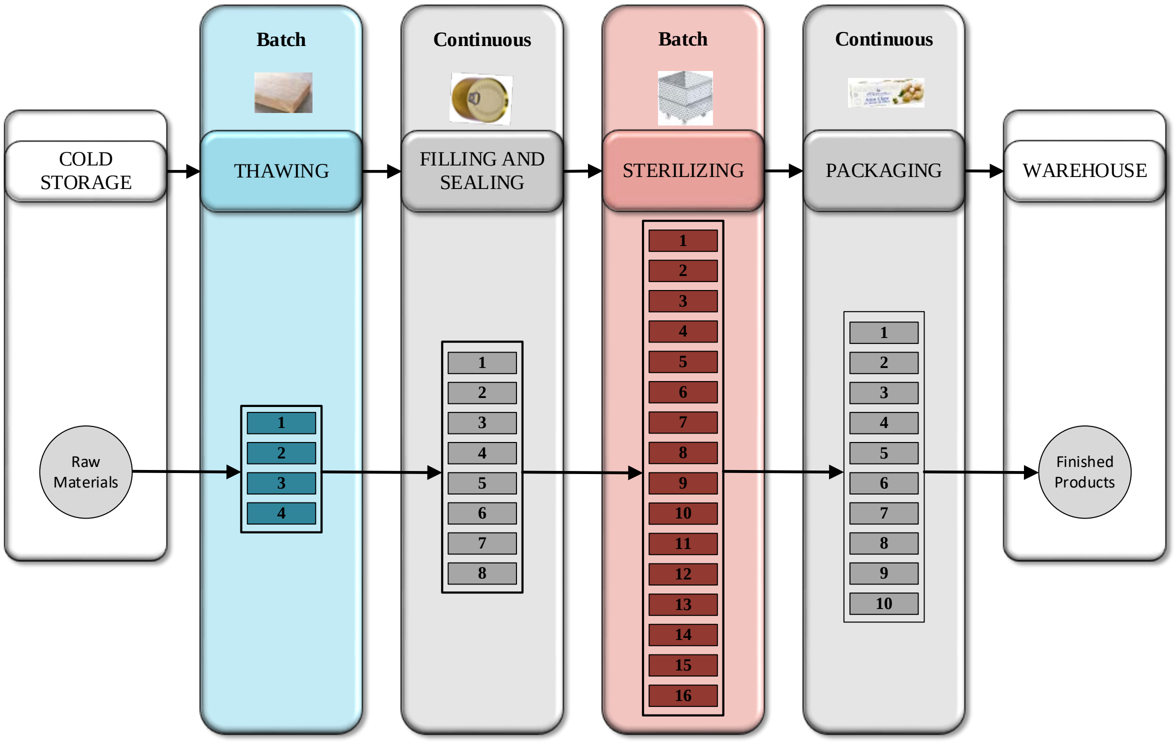

Figure 1. Process Description

The plant can be described as a multiproduct, multistage facility that combines both batch (thawing, sterilizing) and continuous (sealing and filling, packaging) processes with multiple parallel units. In particular, four thawing chambers, eight filling and sealing lines, sixteen sterilizers and ten packaging lines exist, making up a total of 38 available units in the whole production process. Consequently, an extremely large number of involved items, specifically 4 stages, 38 units and more than 100 products, is reported, making the scheduling problem under study extremely complex. Moreover, one should consider that the order-sizes are usually larger than the sterilization chamber's capacity, therefore each order is split into multiple order-batches, thus significantly increasing the total number of items to be scheduled. An important feature of the plant is the high production flexibility. In particular, all batch chambers (thawing-sterilization) are identical, while each sealing and filling and packing line consists of different set of machines. Therefore, each product can be processed by all batch units, but only specific continuous lines, which have different processing rates. Furthermore, the processing time of each stage significantly varies, thus making the efficient synchronization of all processes a difficult task. In summary, the complex mixed batch and continuous process, combined with the number of production units and orders, production flexibility and the absence of clear bottlenecks, results to a computationally exhaustive scheduling problem.

The plant operates from Monday to Friday, however in cases of large weekly demands overtime operation during the weekend is allowed. Therefore, the short-term scheduling horizon varies from 5 to 7 days depending on the case study, whereas all processing units are available 24 hours each day. Most products have a single due date at the end of the scheduling horizon; nonetheless, some exceptions may occur. Full demand satisfaction is a prerequisite and orders have to be delivered on time, so tardiness is not allowed. Due to product quality considerations and space-related limitations, once a product campaign starts in the thawing stage, it must be carried out until the completion of all processing stages. Moreover, a single campaign policy is favored in the plant, therefore order splitting is not possible.

In practice, production schedules are generated manually by the plant engineers. As described in the previous paragraph, the industrial problem under study is of extreme combinatorial complexity. Therefore, it is impossible for the production engineers to consider the weekly integrated scheduling problem of all processing stages even using simple heuristic rules. In an attempt to generate a feasible schedule, they decompose the decision- making process into multiple steps. On Wednesday prior to the week under study, they receive the weekly demand from the ERP system and plan the daily production, based on capacity limitations. They consider the filling and sealing stage as the most critical process, due to the existence of large changeover times. Therefore, a weekly plan for the filling and sealing stage is firstly generated, so that large changeover times are avoided. Afterwards, this plan is thrown over the wall to the department responsible for the packaging stage, which checks whether it is feasible based on the capacities of the packing lines. At this point there is a constant back and forth communication until a final plan is achieved. After settling a weekly plan, a daily schedule is generated, two days prior to the day under examination, separately for the filling and sealing and the packaging stage. For example, on Monday for Wednesday, on Tuesday for Thursday and so on. During the whole procedure, the sterilizers are not considered at all. The basic rationale of the production engineers is that the main reason for reduced productivity is the existence of changeover times, therefore they try to minimize them separately in each stage. This approach is however myopic, since they do not consider at all the synchronization of production between stages and the limitations imposed by the sterilization stage. For instance, they may propose a schedule that requires the operation of more than the available sterilizers. Consequently, the actual schedules vary significantly from the planned ones, thus requiring multiple re-iterations throughout the week.

The complexity of the problem, results into a decision-making approach that lacks efficiency and results to schedules far away from the optimal operation. Main goal of this study is to propose a solution method that could be the core of a computer-aided tool which will automate this process and assist production engineers into proposing feasible and better schedules. Therefore, an optimization-based approach is developed that considers all involved stages that affect the efficiency of the generated schedules. Moreover, all operational, technical and industry-specific constraints are considered to ensure that the generated solutions can be applied in the plant. The proposed MILP-based solution strategy is described in detail in the next chapter.

All data considered are real and provided directly by the plant's computer systems, so they correspond to the industrial reality faced by the schedulers. The demand is provided directly by the plant's Enterprise Resource Planning (ERP) system, while all operational data, e.g. processing rates, changeover times etc., are supplied by the Manufacturing Execution System (MES) installed at the facility. Moreover, MES provides the Overall Equipment Effectiveness (OEE) of all processing lines. This factor represents any deviations from the lines' nominal speeds, due to i) equipment breakdowns, ii) minor stoppages, iii) reduced machine speeds, iv) start-up scrap and v) product scrap and is calculated based on historical data. Incorporating the OEE factor in the scheduling problem, provides a way to consider uncertainties caused by the aforementioned reasons, thus increasing the robustness of the generated schedules. All data are assumed to be deterministic, while resources like manpower, steam, electricity etc. are not considered.

3 Mathematical formulation

In this section the developed strategy to solve the FRINSA production scheduling problem is presented. Initially all the assumptions and considerations of the approach are defined (subsection 3.1). Then, the main modelling concept is demonstrated in subsection 3.2. Furthermore, the solution strategy is thoroughly analyzed in subsection 3.3 and finally a rolling horizon framework is introduced in subsection 3.4.

3.1 Problem statement

The problem under study can be formally stated as follows.

Given:

- A known scheduling horizon *H* divided into a set to time periods $n \in N$.
- A set of continuous processing stages $s \in S$.
- A set of continuous processing lines $j \in J$.
- The multidimensional set $JS_{j,s}$ describing whether a line *j* belongs in a processing stage *s.*
- The mapping set $CL_{j,j'}$,that denotes packaging lines *j* and *j′* that share the same labeler device.
- A set of products $p \in P$ to be processed within the scheduling horizon, with all production related parameters, such as, demand ζ_p, due date τ_p^{ζ}, processing rate in the continuous lines $\tau_{j,p}^{rate}$ and sterilization time τ_p^{ster}.
- The multidimensional set $JP_{j,p}$ denoting which lines are able to process each product *p*
- A set of product batches $b \in B$. This set is required since the order-sizes are usually larger than the capacity of the sterilization chambers.
- A changeover task is required in a continuous line *j* whenever the production is changed between two different products. Every changeover operation requires a specific time $\gamma_{j,p,p'}$.
- The parameters related to the sterilization stage, in particular the capacity of each cart for every product *p*, χ_p, the number of carts that fill up a sterilization chamber χ^{ST} and the number of available sterilizers in the facility v^{ST}.

Determine:

- The allocation of products into lines in every continuous stage $Y_{s,j,p}$.
- The sequencing between products in each line, which is expressed by general $X_{j,p,p'}^{G}$ and/or immediate $X_{j,p,p'}^{I}$ precedence variables, depending on the utilized MILP model.
- The starting $L_{s,p}$ and completion time $C_{s,p}$ for the production of each product *p* in each continuous stage *s*.
- Variable $CR_{p,n}^{ST}$, that denotes whether a product *p* is being sterilized at time period *n* or not.

After extensive discussions with the production's engineers it was decided that two objectives are relevant to the production and must be addressed. In particular, the minimization of makespan and the minimization of the total changeover time.

Two MILP models are presented to efficiently address the scheduling problem of a multistage multiproduct industrial facility. The first is based on the general precedence framework [10], while the second one is inspired by the unit-specific general precedence formulation [26]. Specific characteristics of the production are exploited to formulate aggregated models, that significantly simplify the problem. However, the combinatorial complexity of the examined problem is still prohibitive for the straightforward application of these models using any known solver, such as CPLEX. Therefore, we also investigate a decomposition strategy, that allows for the fast generation of feasible schedules.

In summary, the developed MILP-based solution strategy consists of three main pillars (Figure 2):

- A pre-processing algorithm that translates production orders into batches.
- The mathematical model describing the scheduling problem.
- A decomposition technique that splits the initial problem into tractable solvable subproblems.

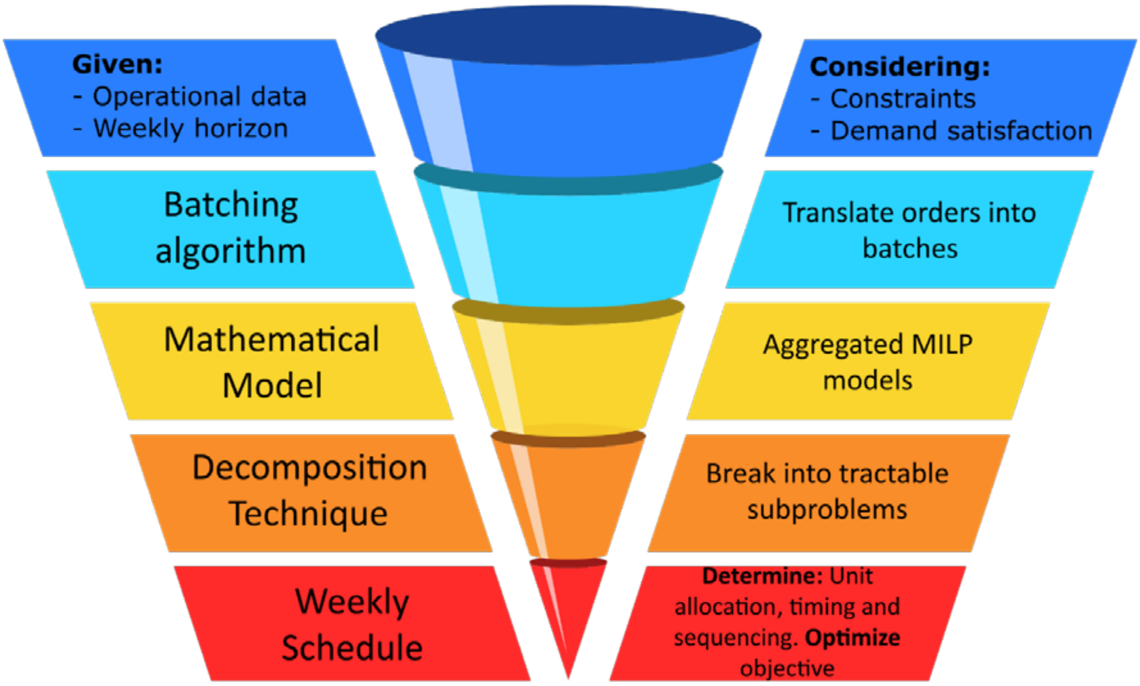

Figure 2. Illustrative description of developed solution strategy

3.2 Conceptual model design

Using MILP-based frameworks to model all processing stages together leads to problems that are intractable with the current computational power. This is mostly due to the large number of involved items, in particular, processing stages, units and products. A common way of addressing complicated problems using low computational times is the simplification of the overall process, by solely focusing on the scheduling a specific stage that constitutes a production bottleneck. Unfortunately, such an assumption cannot be done in this problem, since the production bottleneck shifts according to the demand profile. Therefore, other ways of reducing the problem's complexity, however without generating infeasible schedules, must be investigated. Initially, it was concluded that the thawing stage can be omitted for two main reasons: a) the capacity of the thawing chambers is significantly larger than the rest of the processing lines, b) the defrosted fish can be stored in the chambers for a significant amount of time (more than 24hours). Therefore, any schedule generated by considering all other stages, can be fulfilled by the thawing stage. Despite this reduction the model size remains large. The main source of increased combinatorial complexity is the sterilization process, since in contrast to the rest of the processing stages, each product can be processed by any of the 16 available sterilizers. Unfortunately, we cannot neglect it based on the criteria used for the omission of the thawing stage. However, it is noticed that the scheduling decisions related to the sterilization stage, do not affect the quality of the final schedule. This occurs, since all sterilizers are identical and as such no sequence-dependent setups exist. Therefore, the inclusion of the sterilization stage is a very potential source of degenerate solutions. For instance, let us consider a simple case, in which only two sterilizers *ST1* and *ST2* exist and two products *P1* and *P2* are to be scheduled. Note that alternate allocation decisions i.e. {*P1* → *ST1; P2* → *ST2*} or {*P1* → *ST2; P2* → *ST1*} are equivalent, since the sterilization time for both products is the same using any sterilizer. The same holds for the sequencing decisions. Since no changeovers exist, it does not matter whether *P1* is processed prior to *P2* or vice versa. Based on this observation, the sterilizers can be viewed as a finite renewable resource, similar to e.g. manpower. Therefore, while they are not explicitly modelled, they are indirectly incorporated in the model. Feasibility constraints related to the availability of time and units are imposed. In particular, these constraints ensure, that: a) enough time between the filling and sealing and the packaging process of a product exists, for the required sterilization process and b) that at any time point there are available sterilizers to complete the process. Consequently, the process is reduced to a purely continuous one, consisting of two stages and a number of feasibility constraints for the batch stage in-between. The use of this aggregated approach significantly reduces the combinatorial complexity of the problem at hand. This is illustrated in Figure 3, where all possible production routes for a single product are depicted. It is evident that the suggested aggregated approach decreases substantially the underlying decisions and results into smaller and more efficient MILP models.

An important feature of the aggregated approach is that the proposed problem simplification leads to the modelling of a purely continuous process (filling and sealing and packaging). Moreover, a single campaign policy is utilized, as imposed by the industry. Thus, it is enough to model a single item for every order. In the real process both batch and continuous operations exist. Considering that the sterilization chambers have a capacity smaller than most order-sizes, each order has to be split into numerous batches. Consequently, the suggested approach not only reduces the number of involved units and stages, but also the number of items to be scheduled, thus leading to a considerably smaller model size.

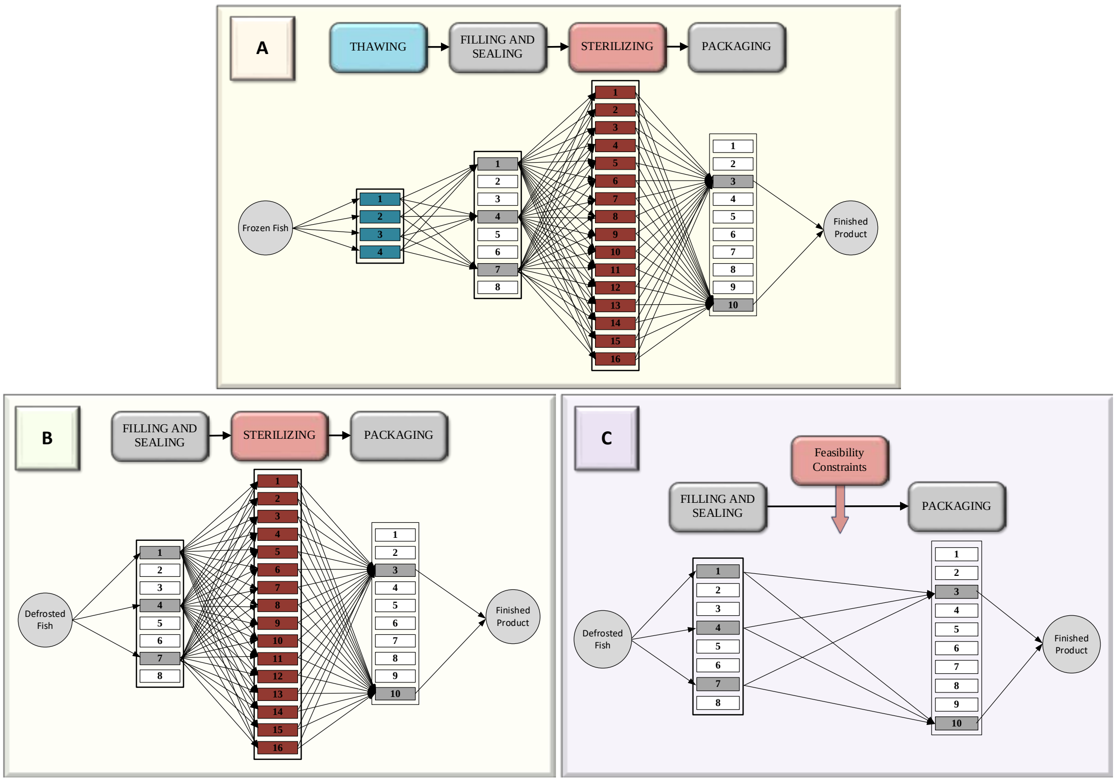

Figure 3. Possible production routes of a single product when a) considering the fully sized problem, b) omitting the thawing stage and c) explicitly modelling of the continuous stages only.

To exploit the benefits of both approaches, a mixed discrete-continuous time representation is used. The size of the problem necessitates the employment of a continuous time representation, since fewer variables are required and smaller, easier solvable, models are generated. However, a known disadvantage of this approach is its inability to efficiently monitor the consumption and/or availability of resources [27]. This is an extremely important feature that must be included in the model, since sterilizers are described as a renewable resource. Therefore, a discrete time grid is employed on top of the continuous one. More specifically, all scheduling decisions related to the continuous stages (filling and sealing, packaging) are modelled in the continuous timeframe, but the feasibility constraints are expressed using the discrete time-grid. The solution quality depends strongly on the duration of the time periods. A finer discretization results to better solutions, and to larger and more difficult to be solved models. Multiple tests have shown that a duration smaller than the fastest sterilization process is a good compromise, since good schedules are generated in low computational times.

In order to further illustrate how time is represented in the developed models, let us consider the simple case illustrated in Figure 4. In this example three products *P1*, *P2* and *P3* are scheduled, over two filling and sealing lines (*FS_1* and *FS_2*) and one packaging line (*P_1*). The continuous time-frame determines, where each product is processed ($Y_{s,j,p}$) in the continuous processes, in what sequence ($X^G_{j,p,p'}$, $X^I_{j,p,p'}$) and exact timing ($L_{s,p}$, $C_{s,p}$). Simultaneously, the timing decisions are mapped on the discrete-time grid. At this point it is assumed that a sterilization process takes place in all discrete-time points between the two continuous stages. This is denoted in the figure by the colored blocks on top of the discrete grid, whose height expresses the number of sterilizers that are required for each product. This number is extracted by the following simple heuristic:

$$Number\ of\ Sterilizers\ (\kappa_P) = \frac{Sterilization\ time\ of\ a\ batch}{Packaging\ time\ of\ a\ batch}$$

, that allows for a constant production in the packaging stage, while using the minimum number of sterilizers. It must be noticed, that while a fully continuous process is modelled, this only occurs due to the employed aggregated approach. In reality, a mixed-discrete continuous process takes place, and each product-lot is split into multiple batches in the sterilization stage. This process characteristic must be considered. In Figure 4 it is clear that the sterilizers are occupied between two time points, one being a little bit later than the start of the filling and sealing stage and the other a little bit earlier than the completion of the packaging stage. This happens, since the sterilization process will only start after processing the first batch in the filling and sealing stage and will stop once the last batch enters the packaging stage. A properly small period duration must be employed, to ensure a good quality of the results. At each time point, the total number of sterilizers used are monitored and bounded not to violate the maximum resource limit.

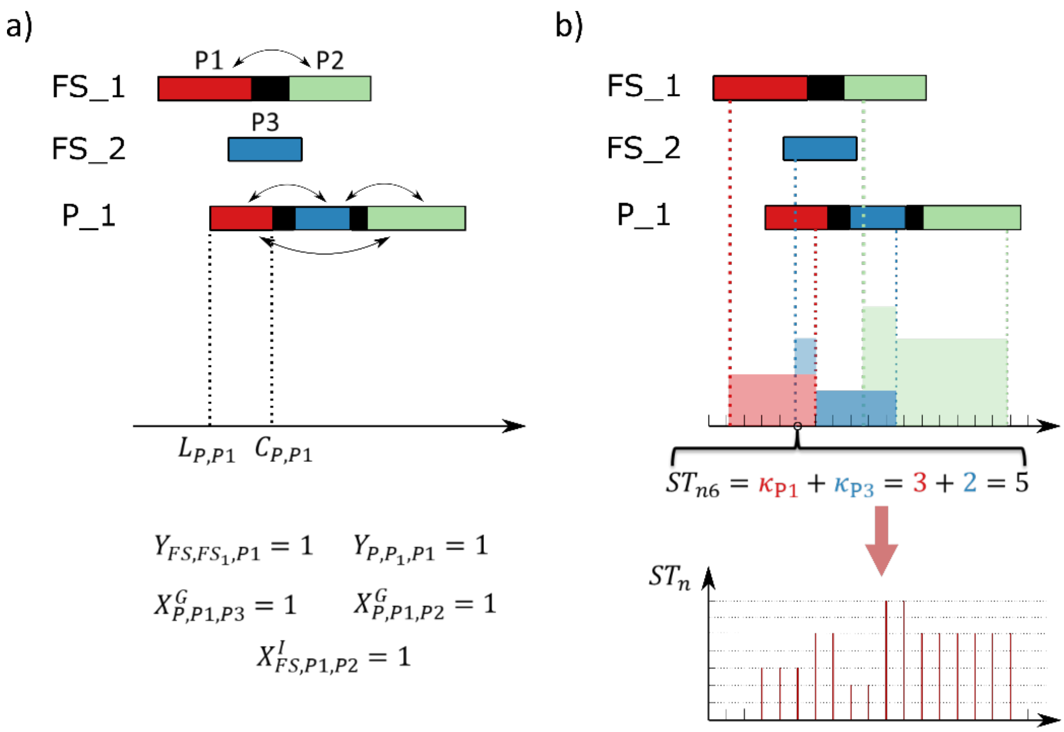

Figure 4. Time representation: a) continuous time-frame; b) discrete time-frame

3.3 Solution strategy

3.3.1 Batching algorithm

The goal of this batching algorithm is to convert the product orders into batches in the sterilization stage to fully satisfy the given demand. Moreover, in this step we calculate the processing time required for the first batch in the filling and sealing stage and the last batch in the packaging stage. These parameters are later required in the mathematical models. In most food industries, such as the one studied in this work, the industrial practice imposes the operation of the intermediate batch processes to their maximum capacity. The maximum utilization of the batch stage allows for high production levels while ensuring minimization of changeovers between products. Thus, the batching problem can be solved a priori. After the completion of the filling and sealing process, the cans are loaded in carts that are pushed into the sterilization chambers. All product orders are at least the size of one full batch; but the capacity of the sterilizers may not be an exact divisor of the order size. Therefore, the last batch of any order may be smaller than the rest. To calculate the necessary batch-related parameters, we employ the following equations:

$$n_p^c = \frac{\zeta_p}{\chi_p} \tag{1}$$

$$n_p^b = \left\lceil \frac{n_p^c}{\chi^{ST}} \right\rceil \tag{2}$$

$$n_p^{Full} = \left\lfloor \frac{n_p^c}{\chi^{ST}} \right\rfloor \tag{3}$$

$$q_p^{FB} = \chi_p \cdot \chi^{ST} \tag{4}$$

$$q_p^{LB} = \begin{cases} \chi_p \cdot \chi^{ST} & , when\ n_p^b = n_p^{Full} \\ (n_p^c - n_p^{FB} \cdot \chi^{ST}) \cdot \chi_p & , when\ n_p^b \neq n_p^{Full} \end{cases} \tag{5}$$

$$\tau_{j,p}^{FB} = \frac{q_p^{FB}}{\tau_{j,p}^{rate}} \qquad \forall j \in (JP_{j,p} \cap JS_{j,s}),\ \ s = "FS" \tag{6}$$

$$\tau_{j,p}^{LB} = \frac{q_p^{LB}}{\tau_{j,p}^{rate}} \qquad \forall j \in (JP_{j,p} \cap JS_{j,s}),\ \ s = "Pack" \tag{7}$$

In equation (1) the total number of required carts for each product is calculated, by dividing product demand ζ_p over the capacity of the carts χ_p, which depends on the size of the products' cans. Equation (2) defines the minimum number of batches required to satisfy demand n_p^b, by dividing the calculated number of carts over number of carts that fill each sterilizer χ^{ST}. Since all sterilizers have the same specifications, this number is constant for all sterilizers and equal to 9. However, the total number of carts for each product may not be exactly divided by this number. Therefore, in equation (3) we also define n_p^{Full}, which is the number of fully utilized batches. Based on that information, the quantity processed in the first and last batch is calculated. The quantity of the first batch of each product order is always equal to the capacity of each cart multiplied by the number of carts that fill a sterilizer, as shown in equation (4). However, the capacity of the last batch depends on whether it is a full batch or a batch of reduced size (Equation 5). Finally, the required processing time for the first batch of each product in each available line of the filling and sealing stage $\tau_{j,p}^{FB}$ and the last batch of each product in every available line of the packaging stage $\tau_{j,p}^{LB}$, is calculated using equations (6) and (7) accordingly.

To better clarify the meaning of these parameters, let us consider an example of an order with a size of 120000 cans of a specific product, with a cart capacity of 5000 cans. In that case, the total amount of required carts is:

$$n_p^c = \frac{\zeta_p}{\chi_p} = \frac{120000}{5000} = 24$$

The number of full batches and total batches are calculated as follows:

$$n_p^{Full} = \left\lfloor \frac{n_p^c}{\chi^{ST}} \right\rfloor = \left\lfloor \frac{24}{9} \right\rfloor = 2$$

$$n_p^b = \left\lceil \frac{n_p^c}{\chi^{ST}} \right\rceil = \left\lceil \frac{24}{9} \right\rceil = 3$$

Furthermore, let us assume that this product can only be processed by one line in the filling and sealing stage and one in the packing stage, with a rate 45000 cans/hour. Therefore, the considered processing times will be

$$\tau_{j,p}^{FB} = \frac{q_p^{FB}}{\tau_{j,p}^{rate}} = \frac{\chi_p \cdot \chi^{ST}}{\tau_{j,p}^{rate}} = \frac{5000 \cdot 9}{45000} = 1 \ \ hour$$

for the first batch in the filling and sealing stage and

$$\tau_{j,p}^{LB} = \frac{q_p^{LB}}{\tau_{j,p}^{rate}} = \frac{(n_p^c - n_p^{FB} \cdot \chi^{ST}) \cdot \chi_p}{\tau_{j,p}^{rate}} = \frac{5000 \cdot (24 - 9 \cdot 2)}{45000} = 40\, minutes$$

for the last batch in the packing stage.

3.3.2 General precedence model (M1)

All models based on the general precedence framework are significant smaller compared to models generated using other continuous MILP frameworks. This is due to the fewer required constraints, making general precedence model attractive for large-scale scheduling problems. For the industrial problem under consideration we propose an MILP model based on the aggregated approach presented in subsection 3.1 and the general precedence framework. Next, we present the developed model, categorizing the constraints according to the type of decisions they subject to.

Allocation constraints. Constraints (8) ensure that all products *p,* to be scheduled within the time horizon of interest, will be processed by a single unit *j* in every stage *s*, using the binary allocation variable $Y_{s,j,p}$. Constraints (9) activates the unit utilization variable V_j. In particular, it states that a unit is used ($V_j = 1$), whenever at least one product is processed by it ($Y_{s,j,p} = 1$).

$$\sum_{j \in (JP_j \cap JS_j)} Y_{s,j,p} = 1 \qquad \forall\, p \in P_p^{in}, s \in S \tag{8}$$

$$V_j \geq Y_{s,j,p} \qquad \forall\, p \in P, j \in (JP_j \cap JS_j), s \in S \tag{9}$$

<u>Timing constraints.</u> Constraints (10) define the connection between the starting $L_{s,p}$ and the completion time $C_{s,p}$ of every product *p* at each stage *s*. Since all orders are completed in a single campaign the required processing time can be simple calculated by dividing the demand by the give processing rate $\tau_{j,p} = \frac{\zeta_p}{\tau_{j,p}^{rate}}$. Constraints (11) state that the completion time of a product *p* in each stage *s* must be larger than the necessary processing time of the product $\tau_{j,p}$ plus the processing times of all products *p′* that are previously processed in the same line ($X_{j,p,p'}^{G}$=1). In the next constraints, the synchronization of production between stages is guaranteed. More specifically, constraints (12) ensure that the starting time of the packing process of a product, is larger than the starting time of the filling and sealing process, plus the processing time of the first batch in the filling and sealing stage $\tau_{j,p}^{FB}$ and the required sterilization time τ_p^{ster}. Similarly, constraints (13) guarantee the synchronization of the completion times of each stage.

$$C_{s,p} = L_{s,p} + \sum_{j \in (JP_{j,p} \cap JS_{j,s})} (\tau_{j,p} \cdot Y_{s,j,p}) \qquad \forall\, p \in P_p^{in}, s \in S \tag{10}$$

$$\begin{aligned} C_{s,p} \geq & \sum_{j \in (JP_j \cap JS_j)} (\tau_{j,p} \cdot Y_{s,j,p}) \\ & + \sum_{j \in (JP_j \cap JP'_j \cap JS_j)} \; \sum_{p' \in P_{p'}^{in}, p \neq p'} (X_{j,p',p}^{G} \cdot \tau_{j,p'}) \end{aligned} \qquad \forall\, p \in P_p^{in}, s \in S \tag{11}$$

$$L_{s,p} \geq L_{s-1,p} + \sum_{j \in (JP_{j,p} \cap JS_{j,s-1})} (\tau_{j,p}^{FB} \cdot Y_{s-1,j,p}) + \tau_p^{ster} \qquad \forall\, p \in P_p^{in}, s = Pack \tag{12}$$

$$C_{s,p} \geq C_{s-1,p} + \sum_{j \in (JP_{j,p} \cap JS_{j,s})} (\tau_{j,p}^{LB} \cdot Y_{s,j,p}) + \tau_p^{ster} \qquad \forall\, p \in P_p^{in}, s = Pack \tag{13}$$

<u>Sequencing constraints.</u> To ensure the proper sequencing of production, big-M constraints (14) and (15) are employed. The big-M parameter is set equal to the duration of the scheduling horizon. According to constraints (14), the starting time of a product *p′* processed after another product *p* in the same unit *j*, is forced to be larger than the starting time of product *p* plus the required processing time and the necessary changeovers $\gamma_{j,p,p'}$. Notice that these constraints are only defined for *p<p′*, therefore the complementary constraint set (15) must be introduced.

$$L_{s,p'} \geq L_{s,p} + \tau_{j,p} \cdot Y_{s,j,p} + \gamma_{j,p,p'} - M \cdot (1 - X_{j,p,p'}^{G}) - M \cdot (2 - Y_{s,j,p} - Y_{s,j,p'})$$

$$\begin{aligned} &\forall\, p \in P_p^{in}, p' \in P_{p'}^{in}, s \in S, p < p', \\ &j \in (JP_{j,p} \cap JP_{j,p'} \cap JS_{j,s}) \end{aligned} \tag{14}$$

$$L_{s,p} \geq L_{s,p'} + \tau_{j,p'} \cdot Y_{s,j,p'} + \gamma_{j,p',p} - M \cdot (X_{j,p,p'}^{G}) - M \cdot (2 - Y_{s,j,p} - Y_{s,j,p'})$$

$$\forall p \in P_p^{in}, p' \in P_{p'}^{in}, s \in S, p < p', \quad j \in (JP_{j,p} \cap JP_{j,p'} \cap JS_{j,s}) \tag{15}$$

Tightening constraints. Constraints (16) impose that a general precedence variable between two products is active, only when both products are processed in the same unit. On the other hand, constraint set (17) guarantees that in case both products *p* and *p'* are processed in the same unit *j*, then product *p* may either be processed before product *p'* or vice versa. In order to satisfy the given due dates τ_p^{ζ}, constraints (18) are introduced.

$$2 \cdot (X^G_{j,p,p'} + X^G_{j,p',p}) \leq Y_{s,j,p} + Y_{s,j,p'} \qquad \forall s \in S, p \in P_p^{in}, p' \in P_{p'}^{in}, p \neq p', \quad j \in (JP_{j,p} \cap JP_{j,p'} \cap JS_{j,s}) \tag{16}$$

$$Y_{s,j,p} + Y_{s,j,p'} \leq V_j + X^G_{j,p,p'} + X^G_{j,p',p} \qquad \forall s \in S, p \in P_p^{in}, p' \in P_{p'}^{in}, p \neq p', \quad j \in (JP_{j,p} \cap JP_{j,p'} \cap JS_{j,s}) \tag{17}$$

$$C_{s,p} \leq \tau_p^{\zeta} \qquad \forall s \in S, p \in P_p^{in} \tag{18}$$

Sterilization feasibility constraints. Constraints (19) - (22) utilize a discrete-time grid to enforce the sterilization stage related feasibility constraints. The auxiliary binary variables $X^{ST}_{p,n}$ and $Z^{ST}_{p,n}$ are introduced to define the binary variable $CR^{ST}_{p,n}$ that is activated when a sterilization process occurs for a product *p* in time period *n*. In particular, constraints (19) enable variable $X^{ST}_{p,n}$ for all time periods after the completion of the first batch in the filling and sealing process, plus a waiting time W_p between the two processes. This variable is bounded to be less than 2 hours, to ensure that no bacterial growth will occur that would spoil the final product. The exact time of each time period is calculated by the term $\delta \cdot n$,with δ being the duration of each time period. Thus, constraints (19) define the beginning of a sterilization process for a product *p*. Similarly, constraints (20) set the completion of the sterilization process, by activating the corresponding variable $Z^{ST}_{p,n}$, for all time periods before the time point defined by the completion of the filling and sealing process, plus the waiting time and the required sterilization time. It is assumed that the waiting time for both the first and last batch are equal, since defining two separate variables does not affect the quality of the solution, while the size of the model is further increased. Constraints (21) impose that a sterilization process for product *p* $(CR^{ST}_{p,n} = 1)$ takes place for those time periods *n*, that both $X^{ST}_{p,n}$ (the process starts before *n*) and $Z^{ST}_{p,n}$ (the process finishes after *n*) are activated. Figure 5 illustrates graphically the role of each variable in the feasibility constraints.

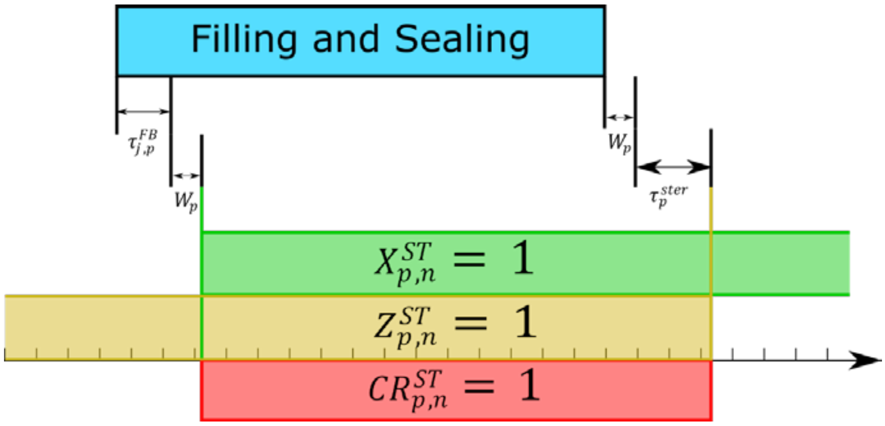

Figure 5. Explanation of binary variables introduced for the sterilization stage constraints

Finally, constraints (22) impose the resource capacity limitations for the sterilization stage. The number of sterilizers used for each product is defined by parameter κ_p, that is only enabled when a sterilization process occurs for this product. It is ensured that at each time point the total number of used sterilizers is less than the available resource v^{ST}.

$$X_{p,n}^{ST} \geq \frac{\delta \cdot n - L_{s,p} - W_p - \sum_{j \in JP_{j,p}} \tau_{j,p}^{FB} \cdot Y_{s,j,p}}{M} \qquad \forall\, p \in P_p^{in}, n \in N \qquad (19)$$

$$Z_{p,n}^{ST} \geq \frac{C_{s,p} + \tau_p^{ster} + W_p - \delta \cdot n}{M} \qquad \forall\, p \in P_p^{in}, n \in N \qquad (20)$$

$$CR_{p,n}^{ST} \geq X_{p,n}^{ST} + Z_{p,n}^{ST} - 1 \qquad \forall\, p \in P_p^{in}, n \in N \qquad (21)$$

$$\sum_{p \in P_p^{in}} (CR_{p,n}^{ST} \cdot \kappa_p) \leq v^{ST} \qquad \forall\, n \in N \qquad (22)$$

<u>Labeler constraints.</u> A significant resource limitation in the industrial facility is the utilization of a single labeler machine in couples of packaging units. Hence, these units cannot operate simultaneously. These design constraints must be considered to ensure the generation of feasible schedules. Therefore, we employ constraints (23) - (28), which were first proposed by Kopanos et al., (2011c). The global sequencing variables $X_{j',p',j,p}^{L}$ are introduced for each pair of products *p'* and *p* that are assigned to different packaging units sharing the same labeler. Constraints (23) and (24) impose that variables $X_{j',p',j,p}^{L}$ are activated when product *p'* starts in unit *j'* before or exactly at the same time that product *p* starts being processed in another unit *j*. In constraints (25) a very small number λ is added to cope with the special case of two products starting at the same time. Auxiliary variables $Z_{j',p',j,p}^{L}$ are active whenever product *p'* is completed in unit *j'* after the starting time of product *p* in another unit j, as constraints (26) state. Finally, binary variables $CR_{j',p',j,p}^{L}$ are added, which denotes that the production of *p'* in *j'* overlaps the one of *p* in unit *j*. As imposed by constraints (27), the variables are active only when both auxiliary variables are equal to one. Finally, constraints

(28) do not allow any overlap in the production of products in lines sharing the same labeler machine.

$$L_{s,p'} - L_{s,p} \le M \cdot (1 - X^{L}_{j',p',j,p}) + M \cdot (2 - Y_{s,j,p} - Y_{s,j',p'}) \qquad \forall\, p \in P^{in}_{p}, p' \in P^{in}_{p'}, p < p', \; j \in (JP_{j,p} \cap CL_{j,j'}), j' \in JP_{j',p'} \tag{23}$$

$$L_{s,p} - L_{s,p'} \le M \cdot X^{L}_{j',p',j,p} + M \cdot (2 - Y_{s,j,p} - Y_{s,j',p'}) \qquad \forall\, p \in P^{in}_{p}, p' \in P^{in}_{p'}, p < p', \; j \in (JP_{j,p} \cap CL_{j,j'}), j' \in JP_{j',p'} \tag{24}$$

$$L_{s,p} - L_{s,p'} + \lambda \le M \cdot X^{L}_{j',p',j,p} + M \cdot (2 - Y_{s,j,p} - Y_{s,j',p'}) \qquad \forall\, p \in P^{in}_{p}, p' \in P^{in}_{p'}, p > p', \; j \in (JP_{j,p} \cap CL_{j,j'}), j' \in JP_{j',p'} \tag{25}$$

$$C_{s,p'} - L_{s,p} \le M \cdot Z^{L}_{j',p',j,p} + M \cdot (2 - Y_{s,j,p} - Y_{s,j',p'}) \qquad \forall\, p \in P^{in}_{p}, p' \in P^{in}_{p'}, \; j \in (JP_{j,p} \cap CL_{j,j'}), j' \in JP_{j',p'} \tag{26}$$

$$CR^{L}_{j',p',j,p} \ge Z^{L}_{j',p',j,p} + X^{L}_{j',p',j,p} - 1 \qquad \forall\, p \in P^{in}_{p}, p' \in P^{in}_{p'}, \; j \in (JP_{j,p} \cap CL_{j,j'}), j' \in JP_{j',p'} \tag{27}$$

$$\sum_{p' \in P^{in}_{p}, p' \neq p} \; \sum_{j' \in (JP_{j',p'} \cap CL_{j,j'})} CR^{L}_{j',p',j,p} \le 0 \qquad \forall\, p \in P^{in}_{p}, j \in JP_{j,p} \tag{28}$$

<u>Cleaning constraints.</u> Due to health-related regulations, the filling and sealing lines must be cleaned at most after 30 hours of operation. In order to take into account these production considerations, we introduce a set of dummy products M_p. Each one of these products denote a cleaning task in each filling and sealing line, with a duration of 2 hours. Since 8 filling and sealing lines exist in the facility and the considered scheduling horizon is weekly, a total of 32 dummy products are introduced. Constraints (29) – (31) are employed to incorporate the cleaning considerations in the model. In particular, constraints (29) impose that the completion of the first cleaning task in each line (FM_M) will be bounded between 10 and 32 hours after the start of the scheduling horizon. Furthermore, constraints (30) ensure that the time elapsed between two cleaning tasks is no more that the allowed 32 hours, while constraints (31) guarantee that at most 32 hours will elapse between the completion of the final cleaning task in each line (LM_M) and the completion of processing any product in the same line.

$$10 \le C_{s,FM} \le 32 \qquad \forall\, FM \in P^{in}_{FM}, s = 'SF' \tag{29}$$

$$C_{s,M} - C_{s,M'} \leq 32 + M \cdot (2 - Y_{s,j,M} - Y_{s,j,M'}) \qquad \begin{array}{l} \forall M \in P_M^{in}, M' \in P_{M'}^{in}, M' = M + 1, \\ j \in (JP_{j,M} \cap JP_{j,M'} \cap SJ_{s,j}), s = 'SF' \end{array} \quad (30)$$

$$C_{s,p} - C_{s,M} \leq 32 + M \cdot (2 - Y_{s,j,p} - Y_{s,j,M}) \qquad \begin{array}{l} \forall P \in P_p^{in}, M \in P_M^{in}, \\ j \in (JP_{j,M} \cap JP_{j,p} \cap SJ_{s,j}), s = 'SF' \end{array} \quad (31)$$

<u>Objective.</u> The main goal of this model is the minimization of the total production makespan $Cmax$.

$$Cmax \geq C_{s,p} \qquad \forall p \in P_p^{in}, s = "Pack" \quad (32)$$

3.3.3 Unit-specific General Precedence Model (M2)

Despite their computational prowess, general precedence models cannot be used when changeover minimization is the main overarching goal of the scheduling problem. To consider this objective, a unit-specific general precedence model is developed. This model (M2) is very similar to the previously presented M1 model, sharing most constraints, with the main difference being the introduction of immediate precedence variables $X^I_{j,p,p'}$. More specifically, model M2 consists of constraints (8) – (10), (12) – (31) and the following:

$$C_{s,p} \geq \sum_{j \in (JP_j \cap JS_j)} (\tau_{j,p} \cdot Y_{s,j,p}) + \sum_{j \in (JP_j \cap JP'_j \cap JS_j)} \sum_{p' \in P_{p'}^{in}, p \neq p'} (X^I_{j,p',p} \cdot \gamma_{j,p',p} + X^G_{j,p',p} \cdot \tau_{j,p'}) \qquad \forall p \in P_p^{in}, s \in S \quad (33)$$

$$L_{s,p'} \geq L_{s,p} + \tau_{j,p} \cdot Y_{s,j,p} + \gamma_{j,p,p'} \cdot X^I_{j,p,p'} - M \cdot (1 - X^G_{j,p,p'}) \qquad \begin{array}{l} \forall p \in P_p^{in}, p' \in P_{p'}^{in}, s \in S, p \neq p', \\ j \in (JP_{j,p} \cap JP_{j,p'} \cap JS_{j,s}) \end{array} \quad (34)$$

$$\sum_{p \in (P_p^{in} \cap JP_{j,p})} \sum_{p' \in (P_p^{in} \cap JP_{j,p}), p' \neq p} X^I_{j,p,p'} + V_j = \sum_{p \in (P_p^{in} \cap JP_{j,p})} Y_{s,j,p} \qquad \forall s \in S, j \in JS_{j,s} \quad (35)$$

$$\sum_{p' \in (P_{p'}^{in} \cap JP_{j,p'}), p' \neq p} X^I_{j,p,p'} \leq Y_{s,j,p} \qquad \forall s \in S, j \in JS_{j,s}, p \in P_p^{in} \quad (36)$$

$$\sum_{p' \in (P_{p'}^{in} \cap JP_{j,p'}), p' \neq p} X^I_{j,p',p} \leq Y_{s,j,p} \qquad \forall s \in S, j \in JS_{j,s}, p \in P_p^{in} \quad (37)$$

$$X^I_{j,p,p'} \leq X^G_{j,p,p'} \qquad \begin{array}{l} \forall p \in P_p^{in}, p' \in P_{p'}^{in}, p \neq p', \\ j \in (JP_{j,p} \cap JP_{j,p'}) \end{array} \quad (38)$$

$$CH = \sum_{j \in (JP_{j,p} \cap JP_{j',p'})} \sum_{p \in P_p^{in}} \sum_{p' \in P_{p'}^{in}, p \neq p'} (\gamma_{j,p,p'} \cdot X_{j,p,p'}^{I}) \tag{39}$$

Constraints (33) constitute an alteration of constraints (11), since they guarantee that the completion time of a product *p* in stage *s* is larger than the required processing time, plus the processing time of all previously completed products in the same line, plus the changeover between product *p* and its direct predecessor *p'*. In contrast to model M1, a single set of sequencing constraints (34) is required, which forces the starting time of product *p'* to be larger than the starting of product *p* that is processed right before it, plus the processing time of *p* and the required changeover time. Four additional tightening constraints are employed. More specifically, constraints (35) state that the total number of processed products in a unit in each stage must be equal to the sum of enabled immediate precedence variables in that unit plus the unit activation variable. Constraints (36) and (37) impose that at most one product *p'* is processed right before or after *p*. Finally, constraints (38) guarantee that a product *p* can be an immediate predecessor of another product *p'* only if it is also a general predecessor. The objective of model M2 is the minimization of the total changeover time CH.

3.3.4 Decomposition algorithm

The aggregated modelling approach presented in the previous subsection reduces significantly the combinatorial complexity of the problem. However, the direct solution of the MILP model still requires large computational effort, thus resulting in intractable case studies. Moreover, the industry requires the solution of the weekly scheduling problem in a prompt manner. This will allow production engineers to undergo multiple what-if analyses, and promptly encounter any order-related uncertainties, like sudden change in demands, cancellations or arrivals of new orders. Main goal of this study is to generate fast near-optimal schedules, which will be readily available to the decision makers. This is essential in order for the developed strategy to be potentially used as the core of a future computer-aided scheduling tool that will be utilized by the production engineers. Therefore, to satisfy the prerequisites set by the industry a decomposition algorithm is employed, that further reduced the complexity of the optimization problem.

An order-based decomposition algorithm is employed to split the initial problem into smaller subproblems. The final schedule is generated in an iterative manner. In each iteration, only a subset of the original set of product orders $p \in P_p^{in}$ is scheduled. Therefore, the generated MILP models are smaller and can be solved much faster. A characteristic of the developed approach, that strongly affects the quality of the solution, is the insertion policy, which consists of: a) the way products are sorted and b) the number of products optimally scheduled in each iteration. Regarding the first decision, multiple possible sorting algorithms were studied. The best solutions were extracted when a sorting from largest to smallest product order size was chosen. This may not be trivial, but can be easily justified since larger orders occupy more time in the scheduling horizon. So, in case other smaller orders are scheduled first, this may be done in a manner that does not allow for the optimal placement of the larger orders. The second decision is a user-defined parameter (σ). Larger values result to better solutions, since the initial problem is less decomposed, but on the other hand require more computational time. Thus, the value of this parameter must be set as high as possible, but not so large that the computational limitations of the examined case study are not met.

In Figure 6 a schematic representation of the developed solution strategy is presented. The input in this method are the plant data provided by the ERP and MES systems and the insertion policy as defined by the user. In the preprocessing step the orders are sorted according to the preferred insertion policy and then the batching algorithm, calculates all batch related parameters. Then, the scheduled problem is solved through an iterative method. The first σ products are inserted in the aggregated models presented in the previous subsection and the MILP for the specified subproblem area is solved. The selected model depends on the scheduling problem's overarching goal. In particular, for makespan minimization, model M1 is used, while model M2 is employed when changeover minimization is the main objective. Afterwards, the unit allocation and the general precedence variables are fixed for the subproblem area. All other related variables, like the utilization of sterilizers and the completion and starting times for the products already scheduled can be freely adjusted in the next iterations to ensure flexibility and improve final results. Then, the algorithm returns to the initial step of the iterative method and the next set of products are inserted. When all product-orders are considered, the complete schedule is generated.

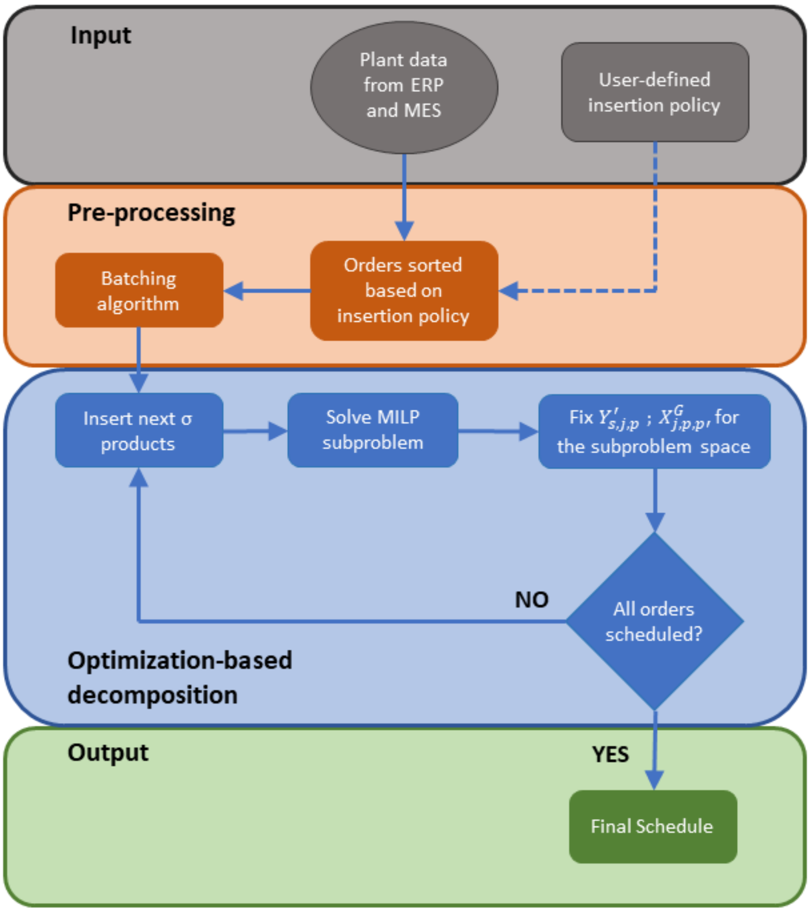

Figure 6. Optimization-based solution strategy

In Figure 7 an illustrative example displaying the allowed and forbidden sequencing decisions, when employing the decomposition algorithm, is presented. In this simple example, we assume that only one unit exists. Two products have been already scheduled, while in the current iteration, just one product is newly inserted. It is illustrated, that the new product can be freely placed anywhere in the scheduling horizon and in any sequence to the others. However, the sequence between the already scheduled products is set and therefore cannot be changed. Notice, that the immediate precedence variables are not fixed, when model M2 is used. Thus, the flexibility in changing decisions in future iterations of the iterative method is increased, which leads to schedules closer to optimality.

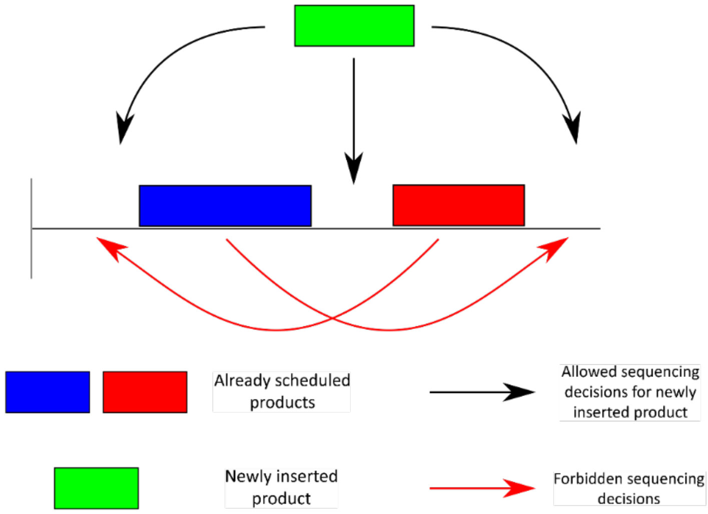

Figure 7. Flexibility of sequencing decisions

3.4 Implementation of a rolling horizon framework

A common issue in food plants is the volatility of product orders as they can be confirmed or changed just prior to their dispatch. An optimization method that does not consider these demand fluctuations, may result in suboptimal or even infeasible production schedules. In such cases a reactive scheduling approach can be applied utilizing the aforementioned MILP models in the context of a rolling horizon algorithm, to ensure that an optimal schedule is generated after the arrival of new information.

The implementation of the algorithm requires the introduction of two new subsets T_p and T_c. T_p corresponds to the prediction horizon, which is the time horizon considered by the optimization model at each iteration of the algorithm. In that time horizon order quantities are assumed to be known with some certainty. A typical prediction horizon is weekly; thus, it includes five time periods ($|T_p|=5$). This also applies for the case of Frinsa del Noroeste. The second subset, T_c, is the control horizon, which includes the time periods in which the decisions provided by the optimization problem are applied. A usual control horizon includes one time period ($|T_c|=1$). Notice that the initial state of the plant in a given prediction horizon $T_{p,h}$ is equal to the final state of the plant in the previous control horizon $T_{c,h-1}$. At each discrete time instant (end of production day) the optimization model receives new information regarding the demand and the state of the plant (see figure 8).

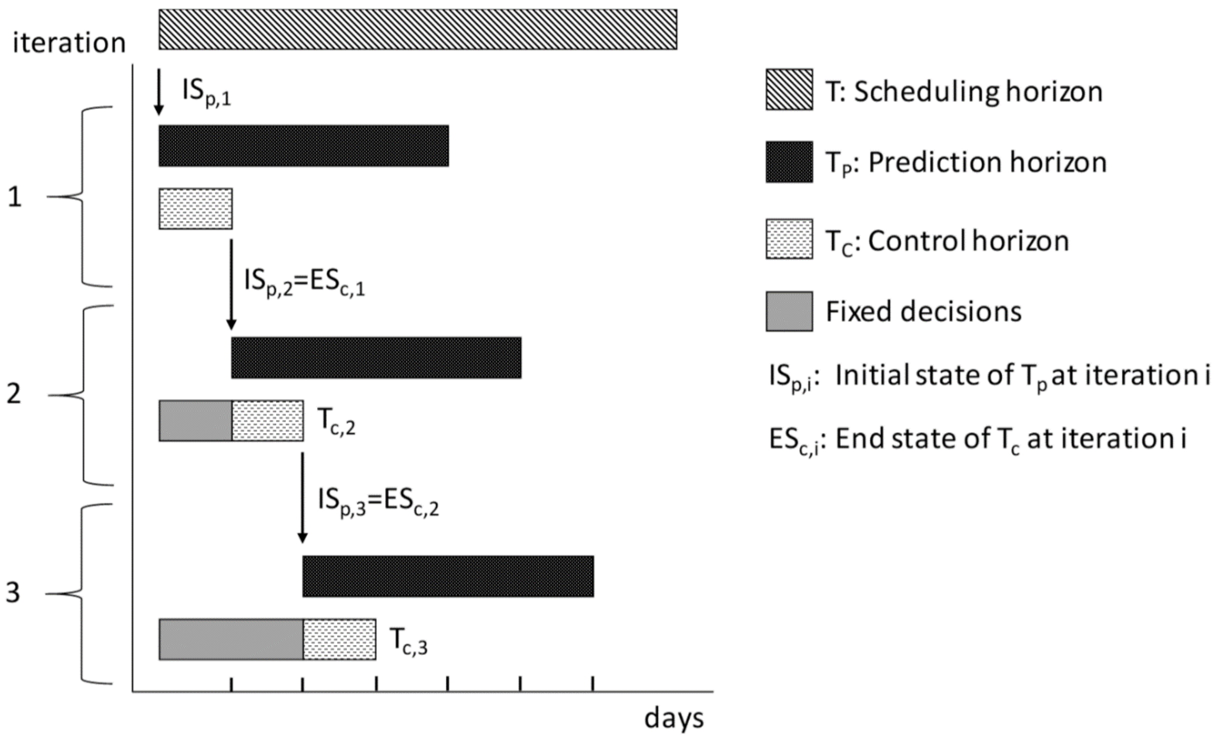

Figure 8. Rescheduling via a rolling horizon approach

Assume an example where the complete scheduling horizon is $|T|=7$ with time periods $T=\{n1,\ldots,n7\}$, $|T_p|=5$ with time periods $T_p=\{n1,\ldots,n5\}$ and $|T_c|=1$. This means that the optimization problem will consider time periods *n1* through *n5*, but the solution (lot-sizes, unit allocation, timing and sequencing of tasks) will be implemented only for *n1*, so $T_c=\{n1\}$. In the next iteration, subsets T_p and T_c are updated so that $T_p= \{n2,\ldots,n6\}$ and $T_c=\{n2\}$. This procedure continues until a production schedule for the complete horizon *T* is extracted. Thus, in the rolling horizon approach the prediction horizon is moving forward in steps of T_c time periods.

This rescheduling strategy can be implemented in our case study in order to handle any changes in demand. Initially, the problem parameters of the plant (e.g. inventories) and the necessary rolling horizon related parameters are defined. Additionally, the number of required iterations to optimize the complete scheduling horizon (*total*) is calculated. Then, the uncertain parameters (product orders) are updated. Thus, order modifications, cancellations and new orders can optimally be dealt with. Afterwards, the proposed MILP is used to solve the scheduling problem for the prediction horizon and the optimal decisions are fixed for the control horizon. At the end of every iteration the state of the plant and the horizon subsets are updated. The algorithm is terminated, when an optimal schedule for the complete time horizon is generated.

4 Results

The applicability and efficiency of the proposed MILP-based solution strategy is illustrated using an industrial case study provided by the Frinsa del Noroeste. In the examined study case, a total of 102 orders have to be efficiently scheduled. All demand-related data is deterministic, however the use of OEE rates increases the robustness of the proposed schedules. The multiproduct, multistage, semi-continuous plant under consideration consists of four processing stages, i.e. thawing, filling and sealing, sterilization and packaging. However, the utilization of the proposed aggregation approach, reduces the optimization problem into two continuous processes (filling and sealing, packaging). Exact schedules are generated only for these stages. However, due to the valid assumptions and the imposed feasibility constraints of the aggregated approach, the proposed schedules will be realized by all stages, without violating any capacity or other limitations. The total number of available sterilizers in the plant is 16 and they are modelled as a common renewable resource. Relevant labeler constraints are also introduced in the packing stage. In particular, the pairs of packaging lines *{P_1; P_2}* and *{P_4; P_5; P_6}* share the same labeling machine, therefore it is forbidden to operate simultaneously. The plant is operating from Monday to Friday, therefore, a 120-hour horizon is considered depending on the problem instance. The implementation of a discrete-time grid requires the discretization of the relevant scheduling horizon into equisized periods. A duration of one hour is chosen for each time period, since the longest sterilization process lasts 82 minutes. Employing a finer discretization, may provide more exact solutions, but the computational cost is prohibitive for the solution of the problem in reasonable CPU times. A challenging prerequisite set by the production engineers is the total computational time required for the generation of near optimal schedules, to be less than 15 minutes. This may be considered as a relatively small CPU time for weekly scheduling, however, such a low solution generation time will allow production engineers to run multiple what-if analyses and re-run the model whenever new information arrives in the plant. Thus, making a future computer-aided tool much more appealing to the production engineers and plant managers.

All models and solution strategies were developed using the GAMS 25.1 interface and all instances were solved in an Intel Core i7 @3.4Gz with 16GB RAM using CPLEX 12.0 [28]. An optimality gap of 3% has been used in each iteration of the solution strategy to ensure the generation of optimal schedules for every MILP subproblem of the decomposition algorithm. A feasible solution was provided in very low computational time (less than 10 minutes). The total makespan of the computed schedule is 112 hours, meaning that the proposed methodology is able to successfully schedule a real week of the FRINSA plant. The Gantt charts of the filling and sealing and packing stage are displayed in figures 9 and 10 accordingly.

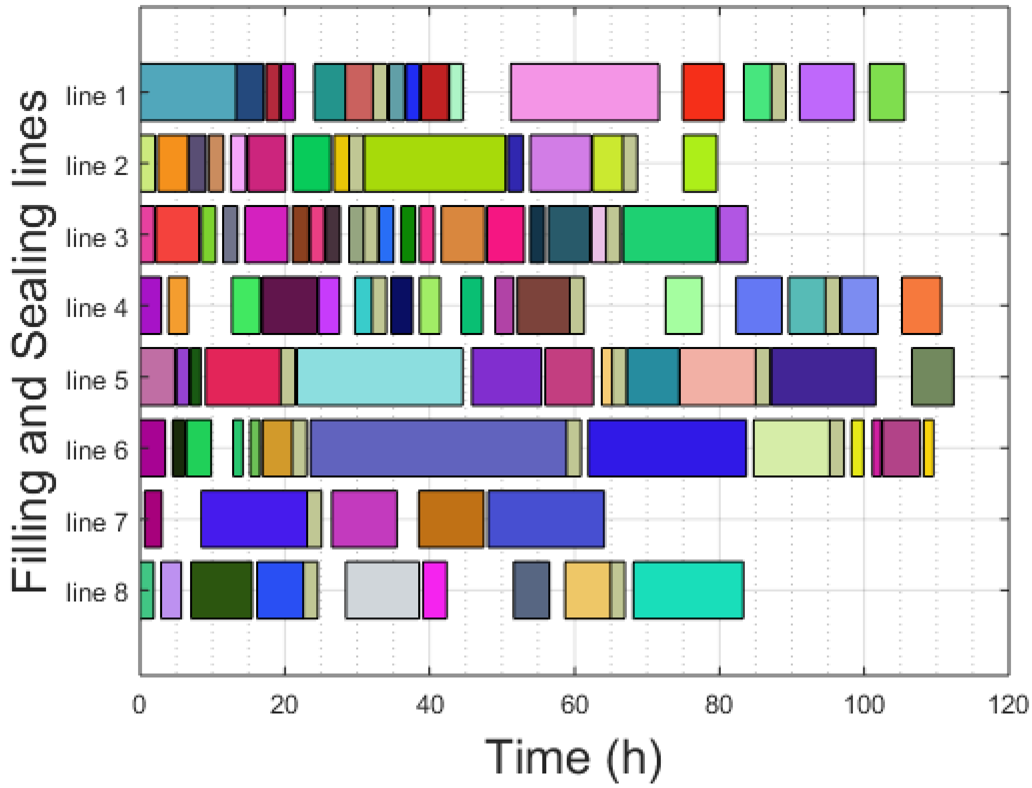

Figure 9. Gantt chart - Filling and Sealing stage

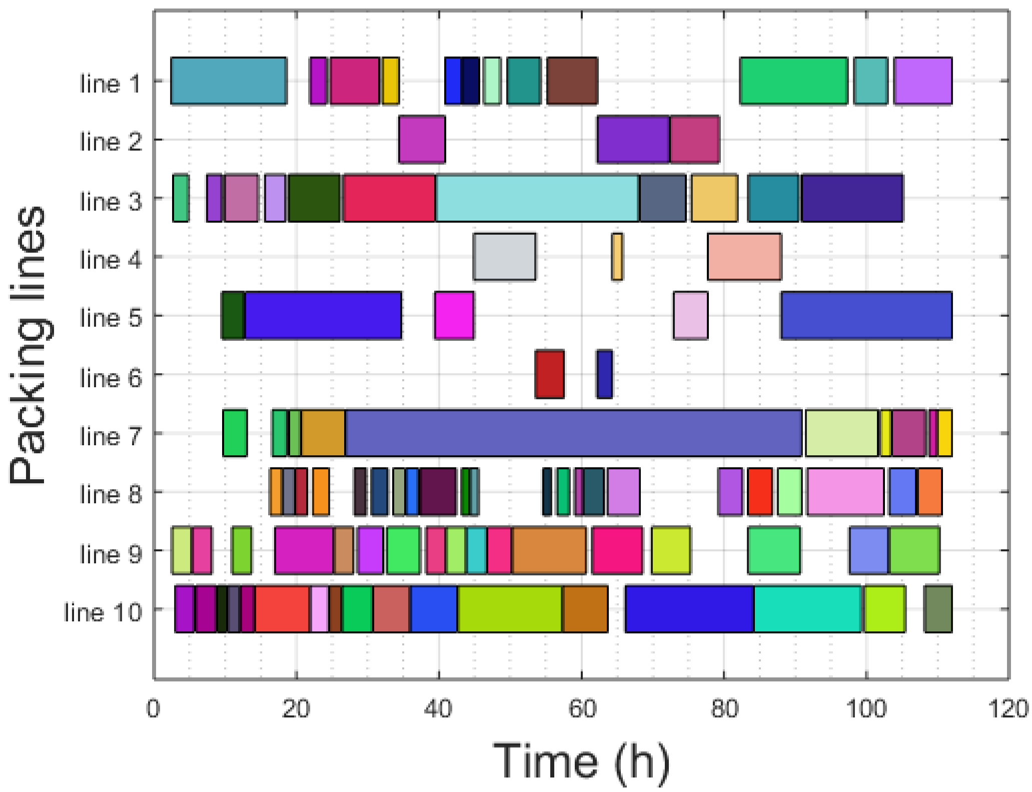

Figure 10. Gantt chart - Packing stage

5 Concluding remarks

In this report a solution strategy for the optimization of the short-term production scheduling of multi-product, multi-purpose and multi-stage production facilities typically met in the process industries such as the food plant of FRINSA is presented. The overall scheduling problem is characterized by a significant combinatorial complexity. More specifically, the industrial facility is described by multiple production stages, each consisting of multiple parallel units, while both continuous and batch processes exist. A large number of products must be processed within a scheduling horizon, resulting to a very large number of decisions to be made. To the best of our knowledge a problem of such complexity has not be successfully solved in a reasonable computational time. In order to efficiently address this problem, an optimization-based approach that consists of a batching algorithm, an MILP model and an order-based decomposition technique, is developed. An aggregated modelling approach, that reduces the complexity of the scheduling problem, is first suggested. More specifically, only the continuous stages are explicitly modelled, while the batch units (sterilizers) are modelled as a renewable resource. Based on this modelling approach, two MILP models are developed, using a mixed discrete-continuous representation which takes into account all operational, design and industry-specific constraints. In the continuous time-frame, a general precedence or a unit-specific general precedence approach is utilized depending on the given objective, while the discrete-time grid is employed to impose the feasibility constraints related to the batch stage. A decomposition algorithm is applied to ensure the solution of the scheduling problem within a given CPU time limit imposed by the industrial partner. In this method, the products to be scheduled are optimized in an iterative manner, according to a user-defined insertion policy. The suggested solution strategy is successfully applied to the real scheduling problem of Frinsa del Noroeste. As a result, optimal weekly schedules can be generated in low computational times. A rolling horizon framework is utilized in combination with the developed solution strategy, in order to efficiently address order-related uncertainties. The proposed optimization framework can be easily extended to address similar large-scale scheduling problems of semi-continuous processes, which are very common in food industries. Moreover, it can be the core of a computer-aided tool that will assist decision-makers to generate fast and near-optimal schedules. Finally, this report illustrates the successful implementation of an optimization-based method for the production scheduling of a real industrial problem, which is a step towards the reduction of the existing gap between industrial reality and research.

6 References

[1] M. L. Pinedo, *Scheduling: Theory, Algorithms and Systems*, 5th ed. Springer, 2016.

[2] I. Harjunkoski, "Deploying scheduling solutions in an industrial environment," *Comput. Chem. Eng.*, vol. 91, pp. 127–135, 2016.

[3] I. Harjunkoski *et al.*, "Scope for industrial applications of production scheduling models and solution methods," *Comput. Chem. Eng.*, vol. 62, pp. 161–193, 2014.

[4] E. Kondili, C. C. Pantelides, and R. W. H. Sargent, "A general algorithm for short-term scheduling of batch operations-I. MILP formulation," *Comput. Chem. Eng.*, vol. 17, no. 2, pp. 211–227, 1993.

[5] C. C. Pantelides, "Unified frameworks for optimal process planning and scheduling.," *Proc. Second Conf. Found. Comput. Aided Process Oper.*, pp. 253–274, 1994.

[6] K. L. Yee and N. Shah, "Improving the efficiency of discrete time scheduling formulation," *Comput. Chem. Eng.*, vol. 22, pp. S403–S410, 1998.

[7] S. Velez and C. T. Maravelias, "Multiple and nonuniform time grids in discrete-time MIP models for chemical production scheduling," *Comput. Chem. Eng.*, vol. 53, pp. 70–85, 2013.

[8] J. M. Pinto and I. E. Grossmann, "A Continuous Time Mixed Integer Linear Programming Model for Short Term Scheduling of Multistage Batch Plants," *Ind. Eng. Chem. Res.*, vol. 34, no. 9, pp. 3037–3051, 1995.

[9] G. Schilling and C. . Pantelides, "A SIMPLE CONTINUOUS-TIME PROCESS SCHEDULING FORMULATION AND A NOVEL SOLUTION ALGORITHM," vol. 20, no. 96, pp. 1221–1226, 1996.

[10] C. A. Méndez, G. P. Henning, and J. Cerdá, "An MILP continuous-time approach to short-term scheduling of resource-constrained multistage flowshop batch facilities," *Comput. Chem. Eng.*, vol. 25, no. 4–6, pp. 701–711, May 2001.

[11] G. M. Kopanos, L. Puigjaner, and M. C. Georgiadis, "Optimal production scheduling and lot-sizing in dairy plants: The yogurt production line," *Ind. Eng. Chem. Res.*, vol. 49, no. 2, pp. 701–718, 2010.

[12] H. Lee and C. T. Maravelias, "Combining the advantages of discrete- and continuous-time scheduling models : Part 1 . Framework and mathematical formulations," *Comput. Chem. Eng.*, vol. 116, pp. 176–190, 2018.

[13] C. A. Méndez, J. Cerdá, I. E. Grossmann, I. Harjunkoski, and M. Fahl, "State-of-the-art review of optimization methods for short-term scheduling of batch processeszz," *Comput. Chem. Eng.*, vol. 30, no. 6–7, pp. 913–946, 2006.

[14] L. G. Papageorgiou and C. C. Pantelides, "Optimal campaign planning/scheduling of multipurpose batch/semicontinuous plants. 1. Mathematical formulation," *Ind. Eng. Chem. Res.*, vol. 35, no. 2, pp. 488–509, 1996.

[15] G. M. Kopanos, L. Puigjaner, and M. C. Georgiadis, "Resource-constrained production planning in semicontinuous food industries," *Comput. Chem. Eng.*, vol. 35, no. 12, pp. 2929–2944, 2011.

[16] N. Susarla, J. Li, and I. A. Karimi, "A Novel Multi-Grid Formulation for Scheduling Semi-Continuous Plants," *Comput. Aided Chem. Eng.*, vol. 31, pp. 1075–1079, Jan. 2012.

[17] P. Baumann and N. Trautmann, "A continuous-time MILP model for short-term scheduling of make-and-pack production processes," *Int. J. Prod. Res.*, vol. 51, no. 6, pp. 1707–1727, 2013.

[18] J. M. Wassick and J. Ferrio, "Extending the resource task network for industrial applications,"

Comput. Chem. Eng., vol. 35, no. 10, pp. 2124–2140, Oct. 2011.

[19] G. P. Georgiadis, A. P. Elekidis, and M. C. Georgiadis, "Optimization-based scheduling for the process industries: From theory to real-life industrial applications," *Processes*, vol. 7, no. 7, 2019.

[20] S. Liu, J. M. Pinto, and L. G. Papageorgiou, "Single-stage scheduling of multiproduct batch plants: An edible-oil deodorizer case study," *Ind. Eng. Chem. Res.*, vol. 49, no. 18, pp. 8657–8669, 2010.

[21] G. M. Kopanos, L. Puigjaner, and C. T. Maravelias, "Production Planning and Scheduling of Parallel Continuous Processes with Product Families," *Ind. Eng. Chem. Res.*, vol. 50, no. 3, pp. 1369–1378, 2011.

[22] T. A. Baldo, M. O. Santos, B. Almada-Lobo, and R. Morabito, "An optimization approach for the lot sizing and scheduling problem in the brewery industry," *Comput. Ind. Eng.*, vol. 72, no. 1, pp. 58–71, 2014.

[23] G. M. Kopanos, L. Puigjaner, and M. C. Georgiadis, "Optimal Production Scheduling and Lot-Sizing in Dairy Plants : The Yogurt Production Line," *Ind. Eng. Chem. Res.*, vol. 49, pp. 701–718, 2010.

[24] G. P. Georgiadis, G. M. Kopanos, A. Karkaris, H. Ksafopoulos, and M. C. Georgiadis, "Optimal Production Scheduling in the Dairy Industries," *Ind. & Eng. Chem. Res.*, vol. 58, no. 16, pp. 6537–6550, Mar. 2019.

[25] Ç. Sel, B. Bilgen, and J. Bloemhof-Ruwaard, "Planning and scheduling of the make-and-pack dairy production under lifetime uncertainty," *Appl. Math. Model.*, vol. 51, pp. 129–144, 2017.

[26] G. M. Kopanos, C. A. Méndez, and L. Puigjaner, "MIP-based decomposition strategies for large-scale scheduling problems in multiproduct multistage batch plants : A benchmark scheduling problem of the pharmaceutical industry," *Eur. J. Oper. Res.*, vol. 207, no. 2, pp. 644–655, 2010.

[27] C. A. Floudas and X. Lin, "Continuous-time versus discrete-time approaches for scheduling of chemical processes: A review," *Comput. Chem. Eng.*, vol. 28, no. 11, pp. 2109–2129, 2004.

[28] A. Brooke, D. Kendrik, A. Meeraus, R. Raman, and R. E. Rosenthal, "GAMS-A User's Guide." GAMS Development Corporation, Washington, DC, 1998.

7 Nomenclature

Indices

$s \in S$	continuous processing stages (filling and sealing - packaging)
$p \in P$	products to be scheduled
$m \in P$	dummy products used to model cleaning tasks in the sealing and filling lines
$j \in J$	units of continuous processing stages (filling and sealing lines – packaging lines)
$b \in B$	batches of products to be scheduled
$n \in N$	time periods

Sets

$JP_{j,p}$	products *p* that a unit *j* can process
$JS_{j,s}$	stage *s* that a unit *j* belongs to
P_p^{in}	subset of products *p* that are inserted in the optimization model
$CL_{j,j'}$	packaging lines *j* and *j'* utilizing the same labeler device

Variables

<u>Binary</u>

$Y_{s,j,p}$	= 1 when product *p* is processed in unit *j* in processing stage *s*
V_j	= 1 when unit *j* is being utilized
$X_{j,p,p'}^{G}$	= 1 when product *p* is processed before product *p'* in unit *j*
$X_{j,p,p'}^{I}$	= 1 when product *p* is processed right before product *p'* in unit *j*
$X_{j',p',j,p}^{L}$	= 1 when product *p'* starts being processed in unit *j'* before or exactly *p* at the time that product *p* starts in unit *j*
$Z_{j',p',j,p}^{L}$	=1 when product *p'* is completed being processed in unit *j'* after the starting time of product *p* in unit *j*
$CR_{j',p',j,p}^{L}$	= 1 when the production of *p'* in *j'* overlaps the one of *p* in another unit *j*
$X_{p,n}^{ST}$	auxiliary variable for $CR_{p,n}^{ST}$
$Z_{p,n}^{ST}$	auxiliary variable for $CR_{p,n}^{ST}$
$CR_{p,n}^{ST}$	sterilization process for product *p* $(CR_{p,n}^{ST} = 1)$ takes place for those time periods *n*

<u>Continuous</u>

$C_{s,p}$	completion time for product *p* in processing stage *s*

$L_{s,p}$ starting time for product *p* in processing stage *s*

W_p waiting time between filling and sealing and sterilization

Objectives

CH total changeover time

C_{max} makespan

Parameters

$\tau_{j,p}$ processing time of each product *p* processed by continuous line *j*

$\tau_{j,p}^{rate}$ processing rate of each product *p* processed by continuous line *j*

$\gamma_{j,p,p'}$ changeover time required between products *p* and *p′* processed by continuous line *j*

$\tau_{j,p}^{FB}$ processing time for the first batch of each product *p* processed by continuous line *j*

$\tau_{j,p}^{LB}$ processing time for the last batch of each product *p* processed by continuous line *j*

τ_p^{ster} sterilization time required for each product *p*

κ_p number of sterilizers for each product order *p* used according to the applied cyclic heuristic

τ_p^{ζ} due date for product *p*

ζ_p demand for product *p*

χ_p capacity of cart when filled with product *p*

χ^{ST} number of carts to fill each sterilizer

n_p^c number of carts used for product *p*

n_p^b number of batches used for product *p*

n_p^{FB} number of full batches used for product *p*

q_p^{FB} quantity of product *p* processed in a full batch

q_p^{LB}	quantity of product *p* processed in the last batch
v^{ST}	number of available sterilizers
M	big-M parameter
λ	a very small number
σ	Number of products scheduled in each iteration of the decomposition algorithm